SOME INSIGHTS IN PATH PLANNING OF SMALL AUTONOMOUS BLIMPS

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Abstract

A blimp is a small airship that has no metal framework and collapses when deflated. In the first part of this paper, kinematics and dynamics modelling of small autonomous non rigid airships is presented. Euler angles and parameters are used in the formulation of this model. In the second part of the paper, path planning is introduced using helices with vertical axes. Motion generation for trim trajectories (helices with constant curvature and torsion) is presented. Then path planning using helices with quadratic curvature and torsion is described, and motion generation on these helices expressed. This motion generation takes into account the dynamics model presented in the first part.

Key-words: Autonomous Airship, path planning, motion generation, Under-actuated systems.

1 Introduction

Unmanned aerial vehicles are a new focus of research, because of their important application potential. They can be divided into three different types: reduced scale fixed wing vehicles (airplanes), rotary wing aircraft (helicopter) or lighter than air (airships). Lighter than air vehicles suit a wide range of applications, ranging from advertising, aerial photography and survey work tasks. They are safe, cost-effective, durable, environmentally benign and simple to operate. Airships offer the advantage of quiet hover with noise levels much lower than helicopters. Unmanned remotely-operated airships have already proved themselves as camera and TV platforms and for specialized scientific tasks. An actual trend is toward autonomous airships.

What makes a vehicle lighter than air is the fact that it uses a lifting gas (i.e. helium or hot air) in order to be lighter than the surrounding air. The principle of Archimedes applies in the air as well as under water. Airships are powered and have some means of controlling their direction. Nowadays non rigid airships or blimps are the most common form. They are basically large gas balloons. Their shape is maintained by their internal overpressure. The most common form of a dirigible is an ellipsoid. It is an aerodynamical profile with good resistance to aerostatics pressures. The only solid parts are the gondola, the set of propeller (a pair of propeller mounted at the gondola and a tail rotor with horizontal axis of rotation) and the tail fins. The membrane material is normally a flexible gas-tight and weather proofed fabric. The envelope holds the helium that makes the blimp lighter than air. Air ballonets, usually two in number, are installed within the gas space and connected to an external air supply pressurized to the required differential above atmospheric. In addition to the lift provided by helium, airships derive aerodynamic lift from the shape of the envelope as it moves through the air. The conventional airship is essentially a low speed vehicle with the power requirement being approximately proportional to the cube of the airspeed. Endurance is one of the primary characteristics of the airship.

The first objective of this paper is to present both kinematics and dynamics models of a small autonomous blimp. This study discusses the motion in 6 degrees of freedom since 6 independent coordinates are necessary to determine the position and orientation of this vehicle. For kinematics, both Euler angles and parameters representations are discussed because the first one is used in dynamics modelling while the second one is used in path planning. For dynamics, a mathematical description of a dirigible flight dynamics must contain the necessary information about aerodynamic, structural and other internal dynamic effects (engine, actuation). The blimp is a member of the family of under-actuated systems because it has fewer inputs than degrees of freedom. In some studies such as [FOS96, HYG00, KHO99, ZHA99, ZIA98], motion is referenced to a system of orthogonal body axes fixed in the airship, with the origin at the center of volume assumed to coincide with the gross center of buoyancy. The model used was written originally for a buoyant underwater vehicle [FOS96, ZIA98]. It was modified later to take into account the specificity of the airship [HYG00, KHO99, ZHA99]. The term buoyancy is used in hydrodynamics while the term static lift is used in aerodynamics. The center of buoyancy is the center of gravity of the displaced fluid. It is the point through which the static lift acts. The center of gravity is the point through which the weight of the object is acting.
In this paper, the origin of the body fixed frame is the center of gravity while in the cited works, it is located in the center of volume.

The second objective of this paper is to generate a desired flight path to be followed by the airship. A mission starts with take-off from the platform where the mast that holds the mooring device of the blimp is mounted. Typically, flight operation modes can be defined as : take-off, cruise, turn, landing, hover... [BES01, CAM99, PAI99, ZHA99]. After the user has defined the goal tasks, the path generator then determines a path for the vehicle that is a trajectory in space. In the first instance, the trajectories considered are trimming or equilibrium trajectories. The general condition for trim requires that the rate of change of the magnitude of the velocity vector is identically zero, in the body fixed frame. Then, the generalized vertical axis helices are presented. Their characteristic is that their curvature and torsion may have a quadratic variation versus the curvilinear abscissa. We deal with directed curves for path planning. The particular case where the rotational axis n is parallel to the vector T of the Frenet Serret frame is studied. Finally the proposed motion generation is presented for a vehicle moving on this path. The problem of trajectory generation is formulated as an optimization problem. This motion generation takes into account in the first instance constraints on velocity and acceleration then more realistic constraints on the two types of inputs : thrusts and tilt angle. The minimum time problem is formulated then solved numerically.

The paper is organized as follows. Modelling is the subject of the second section while the trim trajectories are studied in section three. In section four, path planning using vertical axis helices with quadratic curvature and torsion and motion generation on these helices are presented. Finally, some conclusions and perspectives are presented in the last section.

2 Airship dynamic modelling

2.1 Kinematics

A general spatial displacement of a rigid body consists of a finite rotation about a spatial axis and a finite translation along some vector. The rotational and translational axes in general need not be related to each other. It is often easier to describe a spatial displacement as a combination of a rotation and a translation motions, where the two axes are not related. However, the combined effect of the two partial transformations (i.e rotation, translation about their respective axes) can be expressed as an equivalent unique screw displacement, where the rotational and translational axes coincide. The concept of a screw thus represents an ideal mathematical tool to analyze spatial transformation [ZEF99]. The finite rotation of a rigid body does not obey to the laws of vector addition (in particular commutativity) and as a result the angular velocity of the body cannot be integrated to give the attitude of the body. There are many ways to describe finite rotations. Direction cosines, Rodrigues - Hamilton's (quaternions) variables [BES93, CHE96, FOS96], Euler parameters [WEN91], Euler angles [BES00], can serve as examples. Some of these groups of variables are very close to each other in their nature [ZEF99]. The usual minimal representation of orientation is given by a set of three Euler angles. Assembled with the three position coordinates allow the description of the situation of a rigid body. A (3×3) direction cosine matrix (of Euler rotations) is used to describe the orientation of the body (achieved by 3 successive rotations) with respect to some fixed frame reference.

2.2 Euler angles

Two reference frames are considered in the derivation of the kinematics and dynamics equations of motion. These are the Earth fixed frame R_f and the body fixed frame R_m (figure 1). The position and orientation of the vehicle should be described relative to the inertial reference frame while the linear and angular velocities of the vehicle should be expressed in the body-fixed coordinate system. This formulation is used for underwater vehicles as well [FOS96, ZIA98]. The origin C of R_m coincides with the center of gravity of the vehicle. Its axes $(x_c,\ y_c,\ z_c)$ are the principal axes of symmetry when available. They must form a right handed orthogonal normed frame. The position of the vehicle C in R_f can be described by:

$$\eta_1 = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{1}$$

While the orientation is given by

$$\eta_2 = \begin{pmatrix} \phi \\ \theta \\ \psi \end{pmatrix} \tag{2}$$

with ϕ Roll, θ pitch and ψ Yaw angles (aeronautical angles). The relationship between the Euler angles used and the rotation matrix is[1]

$$R = \begin{pmatrix} c_\psi c_\theta & -s_\psi c_\phi + c_\psi s_\theta s_\phi & s_\psi s_\phi + c_\psi s_\theta c_\phi \\ s_\psi c_\theta & c_\psi c_\phi + s_\psi s_\theta s_\phi & -c_\psi s_\phi + s_\psi s_\theta c_\phi \\ -s_\theta & s_\phi c_\theta & c_\phi c_\theta \end{pmatrix}. \tag{3}$$

$R \in SO_3$ denote the orthogonal rotation matrix that specifies the orientation of the airship frame relative to the inertial reference frame in inertial reference frame coordinates. SO_3 is the special orthogonal group of order 3 which is represented by the set of all (3×3) orthogonal rotation matrices that characteristics are :

$$R^T R = I_3, \quad \text{and } det(R) = 1 \tag{4}$$

where I_3 represents the (3×3) identity matrix.

This description is valid in the region $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$. A singularity of this transformation exists for:

$$\theta = \frac{\pi}{2} + k\pi, \quad k \in \mathcal{Z}$$

If we use the manipulators formulation, at each instant, the configuration (position and orientation) of the airship can be described by an homogeneous transformation matrix

[1] The following shorthand notation for trigonometric function is used:

$$c_\beta := \cos(\beta), \quad s_\beta := \sin(\beta), \quad t_\beta := \tan(\beta).$$

corresponding to the displacement from frame R_f to frame R_m. The set of all such matrices is called SE_3 [SEL96], the special Euclidean group of rigid-body transformations in three dimensions.

$$SE_3 = \left\{ A| \ A = \begin{pmatrix} R & \eta_1 \\ 0_3 & 1 \end{pmatrix}, \ R \in SO_3 \subset \mathcal{R}^{3\times 3}, \ \eta_1 \in \mathcal{R}^3 \right\} \tag{5}$$

SE_3 is a Lie group. $\mathcal{R}^3$ represents the set of (3×1) real vectors and $\mathcal{R}^{3\times 3}$ the set of (3×3) real matrices.

Let's introduce V as the linear velocity of the origin C expressed in the body fixed frame R_m and Ω as the angular velocity expressed in R_m.

$$V = \begin{pmatrix} u \\ v \\ w \end{pmatrix}, \quad \Omega = \begin{pmatrix} p \\ q \\ r \end{pmatrix}$$

The kinematics of the airship can be expressed in the following way :

$$\begin{pmatrix} \dot{\eta}_1 \\ \dot{\eta}_2 \end{pmatrix} = \begin{pmatrix} R & 0_3 \\ 0_3 & J(\eta_2) \end{pmatrix} \begin{pmatrix} V \\ \Omega \end{pmatrix} \tag{6}$$

Where

$$J(\eta_2) = \frac{1}{c_\theta} \begin{pmatrix} c_\theta & s_\phi s_\theta & c_\phi s_\theta \\ 0 & c_\phi c_\theta & -s_\phi c_\theta \\ 0 & s_\phi & c_\phi \end{pmatrix} \tag{7}$$

If we use the metric formulation, the tangent space of SE_3, denoted by se_3 is given by :

$$se_3 = \left\{ \begin{pmatrix} sk(\Omega) & V \\ 0 & 0 \end{pmatrix}, \ sk(\Omega) \in \mathcal{R}^{3\times 3}, \ V \in \mathcal{R}^3 \right\} \tag{8}$$

where $sk(\Omega)$ represents the skew-symmetric matrix of the vector Ω

$$\mathrm{sk}(\Omega) = \begin{pmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{pmatrix}$$

This matrix has the property that for an arbitrary vector $U \in \mathcal{R}^3$

$$sk(\Omega)U = \Omega \times U \tag{9}$$

$\times$: represents the cross vector product in $\mathcal{R}^3$. This tangent space se_3 has the structure of a Lie algebra.

There is, however, a geometric singularity (singularity of the analytical jacobian) in the Euler representation. This problem can be avoided by using a 4 parameter description of the orientation, known as quaternion, which can be used to describe all possible orientations. The quaternion approach uses Euler theorem which states that any rotation of a rigid body can be described by a single rotation about a fixed axis. The advantage of using quaternions is their representation and computational efficiency, apart avoiding singularities.

2.2.1 Euler parameters

The Euler parameters are unit quaternions and are represented by a normalized vector of 4 real numbers. They are one of the minimal set of parameters capable of defining a non singular mapping between the parameters and their corresponding rotation matrix. Taking the vector part of a unit quaternion and normalizing it, we can find the rotational axis and from the scalar part, we can obtain the angle of rotation. The Euler parameters are expressed by the rotation axis n and the rotation angle δ about the axis as follows :

$$\epsilon = \begin{pmatrix} \cos(\frac{\delta}{2}) \\ \sin(\frac{\delta}{2})n \end{pmatrix} = \begin{pmatrix} e_0 \\ e \end{pmatrix} = \begin{pmatrix} e_0 \\ e_1 \\ e_2 \\ e_3 \end{pmatrix} = \begin{pmatrix} \cos(\frac{\delta}{2}) \\ \sin(\frac{\delta}{2})n_x \\ \sin(\frac{\delta}{2})n_y \\ \sin(\frac{\delta}{2})n_z \end{pmatrix}, \quad 0 \leq \delta \leq 2\pi \tag{10}$$

The orientation matrix R is given by:

$$R = \begin{pmatrix} 1-2(e_2^2+e_3^2) & 2(e_1e_2-e_3e_0) & 2(e_1e_3+e_2e_0) \\ 2(e_1e_2+e_3e_0) & 1-2(e_1^2+e_3^2) & 2(e_2e_3-e_1e_0) \\ 2(e_1e_3-e_2e_0) & 2(e_2e_3+e_1e_0) & 1-2(e_1^2+e_2^2) \end{pmatrix} \tag{11}$$

The configuration space $\mathcal{R}^3 \times SO_3$ may be replaced by $\mathcal{R}^3 \times S^3$ where

$$S^3 = \{x \in \mathcal{R}^4 | \; ||x||_2 = 1\}$$

is the unit sphere in $\mathcal{R}^4$ for the Euclidean norm $||.||_2$.

The associated kinematics differential equations of Euler parameters are given by :

$$\begin{aligned} \dot{e} =& \frac{1}{2}(e_0 I_3 + \text{sk}(e))\Omega = \frac{1}{2}(\Omega \times e + e_0\Omega) \\ \dot{e}_0 =& -\frac{1}{2}e^T\Omega \end{aligned} \tag{12}$$

and define a vector field on S^3, the 3 sphere on $\mathcal{R}^4$.

The time derivative of Euler parameters can be determined at any configuration of the body if the angular velocity is given. The kinematics of the airship can be expressed in the following way:

$$\begin{pmatrix} \dot{\eta} \\ \dot{\epsilon} \end{pmatrix} = \begin{pmatrix} R & 0_{3\times3} \\ 0_{4\times3} & J(\epsilon) \end{pmatrix} \begin{pmatrix} V \\ \Omega \end{pmatrix} \tag{13}$$

Where

$$J(\epsilon) = \frac{1}{2}\begin{pmatrix} -e_1 & -e_2 & -e_3 \\ e_0 & -e_3 & e_2 \\ e_3 & e_0 & -e_1 \\ -e_2 & e_1 & e_0 \end{pmatrix} \tag{14}$$

with

$$J(\epsilon)^T J(\epsilon) = \frac{1}{4} I_3 \tag{15}$$

In dynamic modelling, Euler parameters cannot be used in a Lagrangian approach since in this representation, a configuration is defined by 7 parameters. In this paper, Euler angles are used for dynamic modelling while Euler parameters are used for path planning.

2.3 Dynamics

In this section, analytic expressions for the forces and moments on the dirigible are derived. It is advantageous to formulate the equations of motion in a body fixed frame to take advantage of the vehicle's geometrical properties. There are in general two approaches in deriving equations of motion. One is to apply Newton's law and Euler's law which can give some physical insight through the derivation. The other one is more complicated, it provides the linkage between the classical framework and the Lagrangian or Hamiltonian framework. In this paper, applying Newton's laws of motion relating the applied forces and moments to the resulting translational and rotational accelerations assembles the equations of motion for the 6 degrees of freedom. The forces and moments are referred to a system of body-fixed axes, centered at the blimp center of gravity. We will make in the sequel some simplifying assumptions : the earth fixed reference frame is inertial, the gravitational field is constant, the airship is supposed to be a rigid body, meaning that it is well inflated, the aero-elastic effects are ignored, the density of air is supposed to be uniform, and the influence of gust is considered as a continuous disturbance, ignoring its stochastic character [MIL73, TUR73]. The deformations are considered to be negligible. The buoyancy system lifetime will be limited by a number of components and factors. Included is the corrosion of unprotected airship skin, degradation of the airship skin due to thermal cycling and temperature exposure and buoyant gas leakage. High temperature will increase permeability of the airship skin and increase leakage. Introducing all these factors into the dynamic model would result in very complicated partial differential equations.
Here, the dynamics model is defined as the set of ordinary differential equations relying the situation of the vehicle in its position, velocity and acceleration to the control vector. The translational part is separated from the rotational part. As the blimp displays a very large volume, its virtual mass and inertia properties cannot be neglected. The dynamics model is

expressed in the body fixed frame as:

$$\begin{aligned}\dot{X} =& -\Omega \times X + V \\ M\dot{V} =& -\Omega \times MV - \mathrm{diag}(D_v)V + (mg - B)R^T e_z + F_1 + F_2 \\ \dot{R} =& Rsk(\Omega) \\ I\dot{\Omega} =& -\Omega \times I\Omega - \mathrm{diag}(D_\Omega)\Omega + \left(R^T e_z \times \bar{BG}\right) B - F_1 \times \bar{P_1G} - F_2 \times \bar{P_2G}\end{aligned} \tag{16}$$

where

$$X = R^T \eta_1 \tag{17}$$

X represents the position in the body fixed frame.
m: is the mass of the airship, the propellers and actuators,
M: is the (3×3) mass matrix and includes both the airship's actual mass as well as the virtual mass elements associated with the dynamics of buoyant vehicles,
I: is the (3×3) inertia matrix and includes both the airship's actual inertia as well as the virtual inertia elements associated with the dynamics of buoyant vehicles, with respect to G,
$\mathrm{diag}(D_v)$: is a (3×3) aerodynamic forces diagonal matrix,
$\mathrm{diag}(D_\Omega)$: is a (3×3) aerodynamic moments diagonal matrix,
F_1 and F_2: Vectors of the propulsion forces,
$e_z = (0 \ \ 0 \ \ 1)^T$: is a unit vector,
$B = \rho\Delta g$: represents the magnitude of the buoyancy force. Δ is the volume of the envelope, ρ is the difference between the density of the ambient atmosphere ρ_{air} and the density of the helium ρ_{helium} in the envelope, g is the constant gravity acceleration.

$\bar{P_iG}$: represents the position of the ith propeller.
$\bar{BG} = (x_B, \ y_B, \ z_B)$ represents the position of the center of buoyancy with respect to the body fixed frame.
The terms $\Omega \times MV$ and $\Omega \times I\Omega$ show the centrifugal and Coriolis components.

The radiation induced forces and moments can be identified as the sum of three components [FOS96]:

- Added mass due to the inertia of the surrounding fluid,
- Radiation induced potential damping due the energy carried away by the wind,
- Restoring forces due to Archimedes (weight and buoyancy).

Added mass should be understood as pressure - induced forces and moments due to a forced harmonic motion of the body which are proportional to the acceleration of the body. In order to allow the vehicle to pass through the air, the fluid must move aside and then close behind the vehicle. As a consequence, the fluid passage possesses kinetic that it would lack if the vehicle was not in motion. The mass of the dirigible is assumed to be concentrated in the center of gravity

$$M = \begin{pmatrix} m - X_x & -X_y & -X_z \\ -Y_x & m - Y_y & -Y_z \\ -Z_x & -Z_y & m - Z_z \end{pmatrix} \tag{18}$$

where $X_{x.}$, $Y_{y.}$ and $Z_{z.}$ are the virtual mass terms of X, Y, Z axes respectively.

$$I = \begin{pmatrix} I_x - K_x & -I_{xy} & -I_{xz} \\ -I_{yx} & I_y - M_y & -I_{yz} \\ -I_{zx} & -I_{zy} & I_z - N_z \end{pmatrix} \tag{19}$$

K_x, M_y and N_z are the virtual inertia terms of X, Y, Z about GX, GY and GZ axes respectively.

The mass and inertia matrices are positive definite. We will assume that the added mass coefficients are constant. They can be estimated from the inertia ratios and the airship weight and dimension parameters.

The aerodynamic force can be resolved into two component forces, one parallel and the other perpendicular to the direction of motion. Lift is the component of the aerodynamic force perpendicular to the direction of motion and drag is the component opposite to the direction of motion.

$$\mathrm{diag}(D_v) = \mathrm{diag}(-X_u - X_{uu}|u|,\ -Y_v - Y_{vv}|v|,\ -Z_w - Z_{ww}|w|) \tag{20}$$

$$\mathrm{diag}(D_\Omega) = \mathrm{diag}(-L_p - L_{pp}|p|,\ -M_q - M_{qq}|q|,\ -N_r - N_{rr}|r|) \tag{21}$$

For a slow moving object in the air, we can assume a linear relationship between the speed and the drag. The aerodynamic drag of an airship is high. For a typical airship in steady axial flight, we consider that the total aerodynamic drag owes its origin to the bare hull. Since both the gravitational force and the buoyancy force are static, they may be treated together. The gravitational force vector is given by the difference between the airship weight and the buoyancy acting upwards on it:

$$\begin{aligned} (mg - B)R^T e_z &= (mg - B) \begin{pmatrix} -s_\theta \\ c_\theta s_\phi \\ c_\theta c_\phi \end{pmatrix} \\ \text{or } (mg - B)R^T e_z &= (mg - B) \begin{pmatrix} 2(e_1 e_3 - e_2 e_0) \\ 2(e_2 e_3 + e_1 e_0) \\ 1 - 2(e_1^2 + e_2^2) \end{pmatrix} \end{aligned} \tag{22}$$

The gravitational and buoyant moments are given by:

$$B\left(R^T e_z \times \bar{BG}\right) = B \begin{pmatrix} z_B c_\theta s_\phi - y_B c_\theta c_\phi \\ x_B c_\theta c_\phi + z_B s_\theta \\ -y_B s_\theta - x_B c_\theta s_\phi \end{pmatrix}$$

$$\text{or } B\left(R^T e_z \times \bar{BG}\right) = B \begin{pmatrix} 2z_B(e_2e_3 + e_1e_0) - y_B(1 - 2(e_1^2 + e_2^2)) \\ x_B(1 - 2(e_1^2 + e_2^2)) + z_B(e_1e_3 - e_2e_0) \\ 2y_B(e_1e_3 - e_2e_0) - 2x_B(e_2e_3 + e_1e_0) \end{pmatrix} \tag{23}$$

The relationship between the center of gravity and the center of buoyancy is an important parameter. For the airship to remain statically level (aerodynamics and thrust effects are ignored here), the center of gravity should be directly below the center of buoyancy. Any horizontal offset will result in the airship adopting a pitch angle. The vertical separation between these two centers affects the handling characteristics of the airship. The mutual position could be changed using the ballonets.

2.4 Propulsion

Actuators provide the means for maneuvering the airship along its course. A blimp is propelled by thrust. Propellers are designed to exert thrust to drive the airship forward. The most popular propulsion system layout for pressurized non rigid airships is twin ducted propellers mounted either side of the envelope bottom. Another one exists in the tail for torque correction and attitude control. A propeller consists of a certain number of blades rotating about an axis. Six blades per propeller are considered to be the minimum required to produce smooth and continuous thrust without excessive turbulence and inter-blade flow interference. Required power to keep a position against winds increases in proportion to the wind velocity cubed. In aerostatics hovering (floating), its stability is mainly affected by its center of lift in relation to the center of gravity. The blimp's center of gravity can be adjusted to obtain either stable, neutral or unstable conditions. Putting all weight on the top would create a highly unstable blimp with a tendency to roll over in a stable position. In aerodynamics flight, stability can be affected by fins and the general layout of the envelope. Control inertia can be affected by weight distribution, dynamic (static) stability and control power (leverage) available. The best way to obtain a well behaved and easy to control blimp in real time is:

- to reduce control inertia by putting all weight as centered as possible (main concentration close to the center of gravity).
- to increase control power, propellers should be far away from the center of gravity for maximum leverage.
- to maximize stability around 'unwanted' degrees of freedom (mainly the roll angle and velocity), the propellers should be installed such that the 'unwanted ' degrees are not controlled.

If the compensating thrust system is found to be impractical, or only partially effective, the airship must move laterally. A blimp is an under-actuated system with two types of control: forces generated by thrusters and angular inputs controlling the direction of the thrusters (μ is the tilt angle of the propellers) :

$$F_1 = \begin{pmatrix} T_M \sin\mu \\ 0 \\ T_M \cos\mu \end{pmatrix}, \quad F_2 = \begin{pmatrix} 0 \\ T_T \\ 0 \end{pmatrix} \tag{24}$$

where T_M and T_T represent respectively the main and tail thrusters.

Thus in building the non linear six degrees of freedom mathematical model, the additional following assumptions are made:

$$\bar{P_1G} = \begin{pmatrix} 0 \\ 0 \\ P_1^3 \end{pmatrix}, \quad \bar{P_2G} = \begin{pmatrix} -P_2^1 \\ 0 \\ 0 \end{pmatrix}$$

If we consider the plane XZ as a plane of symmetry, the mass and inertia matrices can be written as :

$$M = \begin{pmatrix} m - X_x & 0 & -X_z \\ 0 & m - Y_y & 0 \\ -Z_x & 0 & m - Z_z \end{pmatrix} \tag{25}$$

$$I = \begin{pmatrix} I_x - K_x & 0 & -I_{xz} \\ 0 & I_y - M_y & 0 \\ -I_{xz} & 0 & I_z - N_z \end{pmatrix} \tag{26}$$

If the center of gravity sits below the center of buoyancy, then $\bar{BG} = (0\ 0\ z_B)^T$.

The approach and landing, ground maneuvering and masting phases of flight demand the highest degree of control precision. However, due to the airship's susceptibility to gusts and thermals, its inherent aerodynamic instability and the evaporation of aerodynamic control at low airspeeds, the control system must accomplish these tasks at a time when the control over the airship is greatly reduced. The problems associated with gust sensitivity and aerodynamic instability are fundamental to ellipsoidal airships. One of the current principal challenge is the development of quiet, high efficiency propulsion systems. Any airship designed to achieve near-autonomous mast-docking must have significant mass dedicated to low speed handling. In order to estimate the performance of any airship, it is necessary to estimate the cruise drag coefficient and propulsive efficiency. In general, the thruster force and moment vector will be a complicated function depending on the vehicle linear and angular velocity and the control variables. However, under some assumptions, a linear form can be proposed. In addition, most thruster systems are driven by small DC motors designed for aerial operating conditions. Dynamics of the DC motor should also be included in the dynamics.

3 Trim Trajectories

The fundamentals of flight are in general : straight and level flight (maintenance of selected altitude), ascents and descents, level turns, wind drift correction and ground reference maneuvers. Trim is concerned with the ability to maintain flight equilibrium with controls fixed. A trimmed flight condition is defined as one in which the rate of change (of magnitude) of the aircraft's state vector is zero (in the body-fixed frame) and the resultant of the applied forces and moments is zero. In a trimmed maneuver, the aircraft will be accelerating under the action of non-zero resultant aerodynamic and gravitational forces and moments, these effects will be balanced by effects such as centrifugal and gyroscopic inertial forces and moments. The trim problem is generally formulated as a set of nonlinear algebraic equations.

$$\dot{u} = \dot{v} = \dot{w} = \dot{p} = \dot{q} = \dot{r} = 0 \tag{27}$$

Using Eqn's 6 and 7, the angular velocity can be written as :

$$\begin{aligned} p &= \dot{\phi} - \dot{\psi} sin\theta \\ q &= \dot{\theta} cos\phi + \dot{\psi} sin\phi cos\theta \\ r &= -\dot{\theta} sin\phi + \dot{\psi} cos\phi cos\theta \end{aligned} \tag{28}$$

Differentiating versus time and nullifying these derivatives, we obtain

$$\begin{aligned} p_0 &= -\dot{\psi}_0 \sin\theta_0 \\ q_0 &= \dot{\psi}_0 \cos\theta_0 \sin\phi_0 \\ r_0 &= \dot{\psi}_0 \cos\theta_0 \cos\phi_0 \end{aligned} \tag{29}$$

with one of the solutions given by :

$$\begin{pmatrix} \dot{\phi} = 0 \\ \dot{\theta} = 0 \\ \dot{\psi} = constant = \dot{\psi}_0 \end{pmatrix} \tag{30}$$

Thus

$$\begin{pmatrix} \phi = constant = \phi_0 \\ \theta = constant = \theta_0 \\ \psi = \dot{\psi}_0 t \end{pmatrix} \tag{31}$$

Using Eqn 3, trimming trajectories are characterized by:

$$
\begin{aligned}
\dot{x} =& a_x \cos \dot{\psi}_0 t + b_x \sin \dot{\psi}_0 t \\
\dot{y} =& a_y \cos \dot{\psi}_0 t + b_y \sin \dot{\psi}_0 t \\
\dot{z} =& \dot{z}_0 = -sin\theta_0 u_0 + cos\theta_0 sin\phi_0 v_0 + cos\theta_0 cos\phi_0 w_0
\end{aligned}
$$

where

$$
\begin{aligned}
a_x =& u_0 \cos\theta_0 + v_0 \sin\phi_0 \sin\theta_0 + w_0 \cos\phi_0 \sin\theta_0 \\
b_y =& a_x \\
b_x =& - v_0 \cos\phi_0 + w_0 \sin\phi_0 \\
a_y =& - b_x
\end{aligned}
\tag{32}
$$

Integrating, we obtain

$$
r(s) = \begin{pmatrix} x(s) \\ y(s) \\ z(s) \end{pmatrix}
\tag{33}
$$

with

$$
\begin{aligned}
x =& \frac{a_x}{\dot{\psi}_0} \sin\left(\frac{\dot{\psi}_0}{V_e} s\right) - \frac{b_x}{\dot{\psi}_0} \cos\left(\frac{\dot{\psi}_0}{V_e} s\right) \\
y =& - \frac{b_x}{\dot{\psi}_0} \sin\left(\frac{\dot{\psi}_0}{V_e} s\right) - \frac{a_x}{\dot{\psi}_0} \cos\left(\frac{\dot{\psi}_0}{V_e} s\right) \\
z =& \frac{\dot{z}_0}{V_e} s
\end{aligned}
\tag{34}
$$

where s represents the curvilinear abscissa and we suppose a uniform motion so

$$
s = V_e t = t\sqrt{u_0^2 + v_0^2 + w_0^2}
\tag{35}
$$

The trajectories represented by these equations are helices with constant curvature and torsion (see figure 2). Figure 3 represents the projection of the helix onto the plane $X - Y$. The most general trim condition resembles a spin mode. The spin axis is always directed vertically in the trim. The trim condition can be a turning (about the vertical axis), descending or climbing (assuming constant air density and temperature), side-slipping maneuver at constant speed. More conventional flight conditions such as hover, cruise, auto-rotation or sustained turns are also trims. Motion generation allows to calculate the reference trajectory to be given to the actuators. This reference trajectory definition takes into account the manipulator dynamics.

3.1 Problem formulation

Our objective in this paragraph is to find a value for (u, v, w, ψ) such that an optimization problem is solved where the objective function may be a mixed time-energy function.

Using the dynamic Eqn's 16, 20, 21, 25 and 26, we obtain the following relations.

$$\begin{aligned}
\begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix} &= \begin{pmatrix} T_M \sin\mu \\ T_T \\ T_M \cos\mu \end{pmatrix} \\
&= \begin{pmatrix} (-Z_x u + (m - Z_z)w)q - (m - Y_y)vr - (X_u + X_{uu}|u|)u \\ ((m - X_x)u - X_z w)r - (-Z_x u + (m - Z_z)w)p - (Y_v + Y_{vv}|v|)v \\ (m - Y_y)vp - ((m - X_x)u - X_z w)q - (Z_w + Z_{ww}|w|)w \end{pmatrix} \qquad (36) \\
\begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} &= \begin{pmatrix} 0 \\ -T_M P_1^3 \sin\mu \\ T_T P_2^1 \end{pmatrix} \\
&= -\begin{pmatrix} (-I_{xz}p + (I_z + N_z)r)q - (I_y + M_y)rq - (L_p + L_{pp}|p|)p - Bz_B c_\theta s_\phi \\ ((I_x + K_x)p - I_{xz}r)r - (-I_{xz}p + (I_z + N_z)r)p - (M_q + M_{qq}|q|)q - Bz_B s_\theta \\ (I_y + M_y)pq - ((I_x + K_x)p - I_{xz}r)q - (N_r + N_{rr}|r|)r \end{pmatrix}
\end{aligned}$$

From these 6 relations, we can derive 3 constraints:

$$M_x(u, v, w, \dot{\psi}) = 0 \qquad (37)$$

$$M_y(u, v, w, \dot{\psi}) = P_1^3 F_x(u, v, w, \dot{\psi}) \qquad (38)$$

$$M_z(u, v, w, \dot{\psi}) = -P_2^1 F_y(u, v, w, \dot{\psi}) \qquad (39)$$

The overall problem consists now in determining some variables $(u,\ v,\ w,\ \dot{\psi})$ that minimize the specified objective function: mixed time-energy subject to three equality constraints (dynamics) and inequality constraints (actuators), the weighting parameter being λ. The thrusts T_M and T_T are bounded as well as the angle μ.

$$\min \int_0^{T_f} (\lambda + (1 - \lambda)E)\, d\tau \qquad (40)$$

$$\text{subject to } M_x = 0 \quad M_y = P_1^3 F_x \quad M_z = -P_2^1 F_y$$

$$|T_M| \leq T_{M_{max}} \;\; |T_T| \leq T_{T_{max}} \quad \mu_{min} \leq \mu \leq \mu_{max} \qquad (41)$$

where

$$T_M^2 = F_x^2 + F_z^2, \;\; T_T = F_y, \;\; \mu = atan\frac{F_x}{F_z} \qquad (42)$$

The total time can be expressed as:

$$T_f = \frac{z_f - z_i}{-us_\theta + vc_\theta s_\phi + wc_\theta c_\phi} \qquad (43)$$

while the energy is given by:

$$E = \int_0^{Tf} \left(T_M^2(t) + T_T^2(t)\right) dt \qquad (44)$$

z_f and z_i being respectively the final and initial altitude.

A proposed resolution method is introduced in the following section.

3.2 Resolution of the minimum time problem

Optimization theory gives a solution to the minimum time problem. It is located on the boundary of the admissible set, i.e. the airship moves using maximum actuator capabilities. The resolution will be organized as follows. First, this problem will be solved assuming that the equality constraints are fulfilled and each inequality constraint is saturated. Then the largest value of all the computed times will be taken as the predicted arrival time.
In the first instance, we solve the three equality constraints Eqn's 45, 47-48, this allows us to obtain $(u,\ v,\ w,\ \dot{\psi})$. Using Eqn 37, we find the following second order polynomial equation:

$$a\dot{\psi}^2 + b\dot{\psi}|\dot{\psi}| + c\dot{\psi} + d = 0 \tag{45}$$

where

$$\begin{aligned}
a =& I_{xz} s_\theta c_\theta s_\phi + (I_z + N_z - I_y - M_y) c_\theta^2 s_\phi c_\phi \\
b =& L_{pp} |s_\theta| s_\theta \\
c =& L_p s_\theta \\
d =& -B z_B c_\theta s_\phi
\end{aligned} \tag{46}$$

$\dot{\psi}$ can then be found analytically.
From Eqn.'s 39 , we can deduce

$$w = \frac{M_z - P_2^1((m - X_x)ur + Z_x up - (Y_v + Y_{vv}|v|)v)}{P_2^1(-X_z r - (m - Z_z)p)} \tag{47}$$

From Eqn 38, we find a relationship between u and v.

$$a'u + b'|u|u + c'v + d'|v|v + e' = 0 \tag{48}$$

where

$$\begin{aligned}
a' =& P_1^3 P_2^1 (X_z r + (m - Z_z)p)(Z_x q + X_u) - P_1^3 P_2^1 (m - Z_z)((m - X_x)r + Z_x p)q \\
b' =& P_1^3 X_{uu} (X_z r + (m - Z_z)p) P_2^1 \\
c' =& P_1^3 (m - Y_y) r (X_z r + (m - Z_z)p) P_2^1 + P_1^3 (m - Z_z) q Y_v P_2^1 \\
d' =& P_1^3 (m - Z_x) q Y_{vv} P_2^1 \\
e' =& P_1^3 (m - Z_z) q M_z - M_y (X_z r + (m - Z_z)p) P_2^1
\end{aligned}$$

Let's deal now with inequality constraints. Assuming that each constraint can be saturated, we have to solve a set of five nonlinear equations.

$$T_M^2 = T_{M_{max}}^2, \quad T_T = \pm T_{T_{max}}, \quad \mu = \mu_{min}, \quad \mu = \mu_{max} \tag{49}$$

Solving $T_T = \pm T_{T_{max}},\ \mu = \mu_{min},\ \mu = \mu_{max}$ lead to equations of the form;

$$\sum_{i,j,k,l=0}^{2} C_{ijkl} u^i v^j |u|^k |v|^l = 0$$

Solving $T_M^2 = T_{M_{max}}^2$ lead to an equation of the form;

$$\sum_{i,j,k,l=0}^{4} D_{ijkl} u^i v^j |u|^k |v|^l = 0$$

where the coefficients C_{ijkl} and D_{ijkl} are constants depending on the parameters of the dynamic model and the initial and final configurations. We will take as solution of this optimization problem, the smallest value of u and v satisfying the equality constraints and minimizing the objective function.

4 An example of generalized helices

4.1 Vertical axis helix

Arc length parameterization of a curve describing the path of a rigid body in the space is used to derive the equations of the motion of the system. We will use the Frenet - Serret formulation.

$$T(s) = \frac{dr/d\beta}{|dr/d\beta|},\ \ \frac{dT}{ds} = \kappa N(s),\ \ \frac{dN}{ds} = \tau B(s) - \kappa T,\ \ B(s) = T(s) \times N(s),\ \ \frac{dB}{ds} = -\tau N(s) \tag{50}$$

The curvature $\kappa(s)$ measures how quickly the curve is pulling away from the tangent. The torsion $\tau(s)$ measures how quickly the curve is pulling out of the osculating plane (that is the plane defined by the normal $N(s)$ and the tangent $T(s)$). $(T,\ N,\ B)$ are three unitary orthogonal vectors, forming the so called Frenet - Serret frame. In the trim helix presented in the previous paragraph, both the curvature and the torsion are constant. We propose as a generalization of this helix, helices with quadratic (or linear) curvature and torsion, such that

$$r = \begin{pmatrix} a\cos(\beta(s)) \\ a\sin(\beta(s)) \\ b\beta(s) \end{pmatrix} \tag{51}$$

with

$$\beta(s) = \alpha_3 s^3 + \alpha_2 s^2 + \alpha_1 s + \alpha_0 \tag{52}$$

and the curvature and torsion are respectively given by:

$$\begin{aligned} \kappa(s) =& \frac{a}{\sqrt{a^2+b^2}}(3\alpha_3 s^2 + 2\alpha_2 s + \alpha_1) \\ \tau(s) =& \frac{b}{\sqrt{a^2+b^2}}(3\alpha_3 s^2 + 2\alpha_2 s + \alpha_1) \end{aligned} \tag{53}$$

Given any two continuous functions κ and τ, there exists a unit speed curve that has these functions as its curvature and torsion. The Frenet Serret frame is given by:

$$
\begin{aligned}
T(s) &= \frac{1}{\sqrt{a^2+b^2}} \begin{pmatrix} -a\sin(\beta(s)) \\ a\cos(\beta(s)) \\ b \end{pmatrix}, \quad N(s) = \begin{pmatrix} -\cos(\beta(s)) \\ -\sin(\beta(s)) \\ 0 \end{pmatrix}, \\
B(s) &= \frac{1}{\sqrt{a^2+b^2}} \begin{pmatrix} b\sin(\beta(s)) \\ -b\cos(\beta(s)) \\ a \end{pmatrix}
\end{aligned}
\tag{54}
$$

In the following subsection, we consider the particular case where the vector T of the Frenet Serret frame and the orientational axis n are the same, using the representation of the Euler parameters.

4.2 Path planning with vertical axis helices

The following hypotheses are made.The representation by Euler parameters is used. Let two postures initial $P_i^T = (r_{x_i}\ r_{y_i}\ r_{z_i}\ e_{0_i}\ e_{1_i}\ e_{2_i}\ e_{3_i})$ and final $P_f^T = (r_{x_f}\ r_{y_f}\ r_{z_f}\ e_{0_f}\ e_{1_f}\ e_{2_f}\ e_{3_f})$.

Smooth trajectories are preferred because rolling motion is often not controlled in an airship and large vibrations could be generated by discontinuous curvature or torsion. We are looking for the smoothest directed path joining a given pair $(P_i,\ P_f)$ of configurations, such that $(r_{x_i}\ r_{y_i}\ r_{z_i}) \neq (r_{x_f}\ r_{y_f}\ r_{z_f})$. A directed path segment is said to join P_i and P_f if their end points are $(r_{x_i}\ r_{y_i}\ r_{z_i})$ and $(r_{x_f}\ r_{y_f}\ r_{z_f})$ and the tangential orientation at the end points are $(e_{0_i}\ e_{1_i}\ e_{2_i}\ e_{3_i})$ and $(e_{0_f}\ e_{1_f}\ e_{2_f}\ e_{3_f})$. An helix defined by Eqn.

$$
r = \begin{pmatrix} a(s)\cos(\beta(s)) \\ a(s)\sin(\beta(s)) \\ b(s)\beta(s) \end{pmatrix}
\tag{55}
$$

has a vertical axis if the curvature and the torsion are proportional in each point

$$
\frac{\kappa(s)}{\tau(s)} = const. \quad \forall s \in \mathcal{R}^+
\tag{56}
$$

Proposition 4.1 *P_i (or P_f) belongs to a vertical axis helix Eqn. 51, if*

$$
r_{z_i} = b.atan\frac{r_{y_i}}{r_{x_i}}
$$

where $a = \sqrt{r_{x_i}^2 + r_{y_i}^2}$ *and* $b = \frac{a n_{z_i}}{\sqrt{n_{x_i}^2 + n_{y_i}^2}}$

Proof In the particular case studied in this paper, the configuration is expressed by 7 elements such that

$$P(s) = \begin{pmatrix} a\cos(\beta(s)) \\ a\sin(\beta(s)) \\ b\beta(s) \\ \cos(\frac{\delta}{2}) \\ \frac{-a}{\sqrt{a^2+b^2}}\sin(\frac{\delta}{2})\sin(\beta(s)) \\ \frac{a}{\sqrt{a^2+b^2}}\sin(\frac{\delta}{2})\cos(\beta(s)) \\ \sin(\frac{\delta}{2})\frac{b}{\sqrt{a^2+b^2}} \end{pmatrix} = \begin{pmatrix} r_x \\ r_y \\ r_z \\ \cos(\frac{\delta}{2}) \\ \sin(\frac{\delta}{2})n_x \\ \sin(\frac{\delta}{2})n_y \\ \sin(\frac{\delta}{2})n_z \end{pmatrix}$$

This proposition can be demonstrated with straightforward calculations.

If there exist s_1 and s_2 such that $\kappa(s_1) = \kappa(s_2) = 0$ or $\tau(s_1) = \tau(s_2) = 0$ with $s_1 < s_2$, the points $P(s_1)$ and $P(s_2)$ on the vertical axis helix are inflection points where the sign of the curvature (or the torsion) changes. The best way is to take $\kappa(s) = \tau(s) = 0$ for P_i and P_f. $\triangle$

Proposition 4.2 *Let $(P_i,\ P_f)$ be a pair belonging to a same helix. Let L be the length of the helix*

$$L = (\beta_f - \beta_i)\sqrt{a^2+b^2}, \qquad \textit{with } \beta_i = \frac{r_{z_i}}{b} \textit{ and } \beta_f = \frac{r_{z_f}}{b}$$

then the curvature κ, the torsion τ and the angle β of the portion of the vertical axis helix are given by:

$$\kappa(s) = \frac{6a}{\sqrt{a^2+b^2}}\frac{\beta_f - \beta_i}{L^3}s(L-s),\ \tau(s) = \frac{6b}{\sqrt{a^2+b^2}}\frac{\beta_f - \beta_i}{L^3}s(L-s)$$

and

$$\beta(s) = \frac{\beta_f - \beta_i}{L^3}s^2(-2s+3L) + \beta_i,$$

Proof Let's use initial conditions for β and the curvature (or equivalently the torsion)

$$\beta(s) = \alpha_3 s^3 + \alpha_2 s^2 + \alpha_1 s + \alpha_0 \tag{57}$$

and

$$\kappa(s) = \frac{a}{\sqrt{a^2+b^2}}(3\alpha_3 s^2 + 2\alpha_2 s + \alpha_1) = 0 \quad \text{for} \quad s = 0; \quad s = L$$

Thus, we find four linear equations in four unknowns and the resolution gives:

$$\alpha_0 = \beta_i \quad \alpha_1 = 0 \quad \alpha_2 = 3\frac{\beta_f - \beta_i}{L^2} \quad \alpha_3 = -2\frac{\beta_f - \beta_i}{L^3}$$

To compute the length, the following relationship has been used :

$$L = \int_{P_i}^{P_f} ds = \int_{P_i}^{P_f} \sqrt{dx^2+dy^2+dz^2} = \int_{\beta_i}^{\beta_f} \sqrt{a^2+b^2}d\beta = \sqrt{a^2+b^2}(\beta_f - \beta_i)$$

$\triangle$

Once the path has been calculated in the Earth fixed frame, motion on this helix must be investigated and reference trajectories determined taking into account actuators constraints. To make use of the dynamic model, the path should be transformed first from the Earth fixed frame to the body fixed frame (as the dynamic model is expressed in this frame). Trajectory design can be formulated as an optimization problem. A trajectory is defined as a curve describing the time history of the vehicle displacement. Automated airships must follow a predetermined trajectory such as polynomial functions, trapezoidal profiles and sinusoidal curves, in analogy with terrestrial vehicles. The problem of transferring the vehicle, initially at rest from an arbitrary initial configuration to a desired target configuration also at rest, in minimum time is formulated as an optimal control problem.

4.3 Motion generation

4.3.1 Kinematics constraints

The minimum-time motion generation is solved in this paragraph, taking as feasible limits, purely kinematics constraints on the velocity and acceleration. Often, the structure of the minimum time problem requires that at least one of the actuators is always in saturation, whereas the others adjust their outputs so that constraints on motion are not violated while enabling the vehicle to reach its final desired configuration. Although these results are important theoretically, they are not applicable directly. From an user's point of view, it would be preferable to have a suboptimal but simpler solution to implement. For this purpose, we have chosen, a priori, a polynomial trajectory that coefficients are optimized to get a continuous minimum time motion.
If both velocity $\dot{s}$ and acceleration $\ddot{s}$ are bounded respectively by $\dot{s}_{max}$ and $\ddot{s}_{max}$, we can propose a third order polynomial variation of s versus t :

$$s = \delta_3 t^3 + \delta_2 t^2 + \delta_1 t + \delta_0 \tag{58}$$

If we assume that both initial and final configurations are at rest :

$$s = \frac{L}{T_f^3} t^2(-2t + 3T_f) \tag{59}$$

Differentiating, the maximum value for the velocity is : $\dot{s}_{max} = \frac{3}{2}\frac{L}{T_f}$ while the maximum value for the acceleration is given by : $\ddot{s}_{max} = 6\frac{L}{T_f^2}$
We can propose the following optimization problem :

$$\begin{aligned} & \min T_f \\ \text{subject to} \quad & |\dot{s}| \leq \dot{s}_{max} \quad |\ddot{s}| \leq \ddot{s}_{max} \end{aligned} \tag{60}$$

The classical solution is : $T_f = max(\frac{3}{2}\frac{L}{\dot{s}_{max}}, \sqrt{6\frac{L}{\ddot{s}_{max}}})$
In this way, motion generation is well defined and the reference trajectories as well as the path are easily determined.

4.3.2 Dynamics constraints

A constant bound on the acceleration must represent the global least upper bound of all operations conditions. This implies that the full capabilities of the actuators cannot be utilized if this conventional approach is taken. The efficiency of the autonomous blimp can be increased by considering the dynamics at the motion generation stage.
More realistically, than for the case of kinematics constraints assuming the main and tail thrusts and the tilt angle are bounded, a minimum time cost criterion is considered ;

$$\begin{aligned} &\min T_f \\ \text{subject to} \quad & |T_M| \leq T_{M_{max}} \quad |T_T| \leq T_{T_{max}} \quad \mu_{min} \leq \mu \leq \mu_{max} \end{aligned} \tag{61}$$

Using Eqn(16), we find the following relations.

$$F_1 + F_2 = M\dot{V} + \Omega \times MV + diag(D_V)V - R^T e_Z(mg - B) = \begin{pmatrix} F_x(s,\dot{s}) \\ F_y(s,\dot{s}) \\ F_z(s,\dot{s}) \end{pmatrix} \tag{62}$$

$$-F_1 \times \overline{P_1G} - F_2 \times \overline{P_2G} = I\dot{\Omega} + \Omega \times I\Omega + diag(D_\Omega) - \left(R^T e_z \times \overline{BG}\right) B = \begin{pmatrix} M_x(s,\dot{s}) \\ M_y(s,\dot{s}) \\ M_z(s,\dot{s}) \end{pmatrix} \tag{63}$$

F_x, F_y, F_z, M_x, M_y, M_z depend on mass, inertia, aerodynamic parameters and desired path curvilinear abscissa s and its derivatives.
The following relationships can thus be derived, as in the previous paragraph;

$$\begin{aligned} M_x(s,\dot{s}) &= 0 \\ M_y(s,\dot{s}) &= P_1^3 F_x(s,\dot{s}) \\ M_z(s,\dot{s}) &= -P_2^1 F_y(s,\dot{s}) \end{aligned} \tag{64}$$

and

$$\begin{aligned} T_M^2 &= F_x^2 + F_z^2 \\ T_T &= F_y \\ \mu &= atan\left(\frac{F_x}{F_z}\right) \end{aligned} \tag{65}$$

These three relations (65) introduce three equality constraints. The exact solution of this optimal problem without assuming that trajectories are polynomial is very difficult to obtain.
Assuming as before that both initial and final configurations are at rest (four conditions) and considering the three equality constraints, lead us to propose a sixth order polynomial variation of s versus t :

$$s = \alpha_6 t^6 + \alpha_5 t^5 + \alpha_4 t^4 + \alpha_3 t^3 + \alpha_2 t^2 + \alpha_1 t + \alpha_0 \tag{66}$$

with

$$\begin{aligned} L &= \alpha_6 T_f^6 + \alpha_5 T_f^5 + \alpha_4 T_f^4 + \alpha_3 T_f^3 + \alpha_2 T_f^2 \\ 0 &= 6\alpha_6 T_f^5 + 5\alpha_5 T_f^4 + 4\alpha_4 T_f^3 + 3\alpha_3 T_f^2 + 2\alpha_2 T_f \end{aligned} \tag{67}$$

To solve the optimization problem, the equality constraints are first considered. Three more non linear relations can be found solving Eqn (65). Then, the inequality constraints are supposed to be saturated so five equations have to be solved .

$$\begin{aligned} & F_x^2 + F_z^2 = T_{M_{max}}^2 \\ & F_y = \pm T_{T_{max}} \\ & atan\left(\frac{F_x}{F_z}\right) = \mu_{min} \quad \text{and} \quad atan\left(\frac{F_x}{F_z}\right) = \mu_{max} \end{aligned} \tag{68}$$

The solution to be kept is the largest one which respect every constraint. Due to the strong non linearities, the resolution has to be done numerically.

5 SIMULATION RESULTS

The lighter than air platform used for simulations is the AS200 by Airspeed Airships. It is a remotely piloted airship designed for remote sensing. It is a non rigid 6m long, 1.4m diameter and 8.6 m^3 volume airship equipped with two vectorable engines on the sides of the gondola and 4 control surfaces at the stern. The four stabilizers are externally braced on the full and rudder movement is provided by direct linkage to the servos. Envelope pressure is maintained by air feed from the propellers into the two ballonets located inside the central portion of the hull. These ballonets are self regulating and can be feed from either engine. The engines are standard model aircraft type units. The propellers can be rotated through 120 degrees. During flight the rudder and elevator are used for all movements in pitch and yaw. In addition, the trim function can be used to alter the attitude of the airship in order to obtain level flight or to fly with a positive or negative pitch angle. Rudder and elevator can be moved from -25 to +25 degrees. The maximum velocity is 13m/s and the maximal height is 200m. Climb or dive angles should not exceed 30 degrees, particularly at full throttle.

We present an example of a generalized helix joining two configurations in space.

$$P_i = \begin{pmatrix} \sqrt{2} \\ \sqrt{2} \\ -2.3 \\ 0.866 \\ 0 \\ 0.5 \\ 0.5 \end{pmatrix} \qquad \mathrm{P}_f = \begin{pmatrix} 0 \\ 5 \\ 3.75 \\ 0.866 \\ -0.320 \\ -0.129 \\ 0.353 \end{pmatrix}$$

from the path planning, we find :

$a = 1.927; \quad b = 1.927; \quad L = 8.56m$

Figure 4 presents this generalized helix with a vertical axis where the curvature and the torsion have a quadratic variation versus s, while figures 5 and 6 present respectively the angle β and the curvature versus the curvilinear abscissa s.

5.1 Kinematics constraints

The values of the limitations on velocity and acceleration are the following :
$\dot{s}_{max} = 10m/s \quad \ddot{s}_{max} = 10m/s^2$
From the motion generation, we find
$T_f = 7.16s$
Figure 7 presents the three forces F respectively in the x, y and z directions. Figure 8 and figure 9 show respectively the linear velocity (u, v, w) and the angular velocity (p, q, r).

5.2 Dynamics constraints

The values of the limitations on inputs are the following :
$\mu_{min} = -2\frac{\pi}{3}rad \quad \mu_{max} = 2\frac{\pi}{3}rad \quad T_{M_{max}} = 115N \quad T_{T_{max}} = 100N$
From the motion generation, we find
$T_f = 10s$
Figure 10 presents the three forces F respectively in the x, y and z directions. Figure 11 and figure 12 show respectively the linear velocity (u, v, w) and the angular velocity (p, q, r).
we can verify that the constraints are respected.

6 CONCLUSIONS

Blimps are a highly interesting study object due to their stability properties. The classical theory of airship stability and control is based on a linearized system of differential equations usually obtained by considering small perturbations about a steady flight condition. However, the constraints of staying within the linear flight regime are excessive. The design of advanced control system must take into account the strong non linearities of the dynamic model. In this prospect, in the first part of this paper, we have discussed kinematics and dynamics of a blimp, using Newton Euler approach. A direct generalization of this model is to introduce the effects of the vertical and horizontal control surfaces. In the second part of this paper, we have discussed characterization of some helices for airships. Trimming trajectories have been presented. They consist in helices with constant curvature and torsion. When specifying a trajectory, the physical limits of the system must be taken into account. For trim flights, we propose a motion generation problem by minimizing the travelling time, given realistic constraints, the generated forces and the tilt angle. Then, generalized helices with vertical axes have been introduced where the curvature and the torsion have a quadratic variation versus the curvilinear abscissa. We are actually investigating how to rejoin any two situations (position and orientation) in space. Is one helix enough or do we need more than one? The path planning and motion generation topics are still a very active research area.

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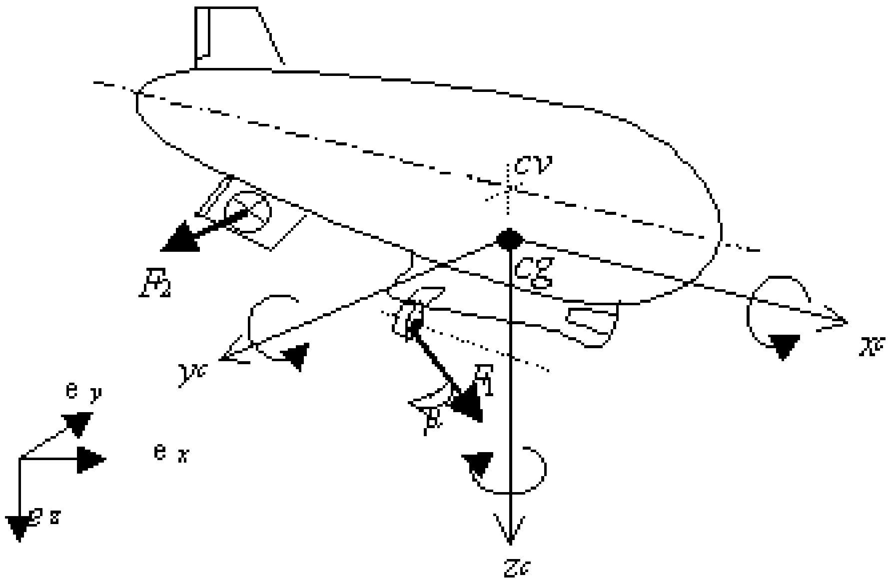

Figure 1: Frames of the blimp

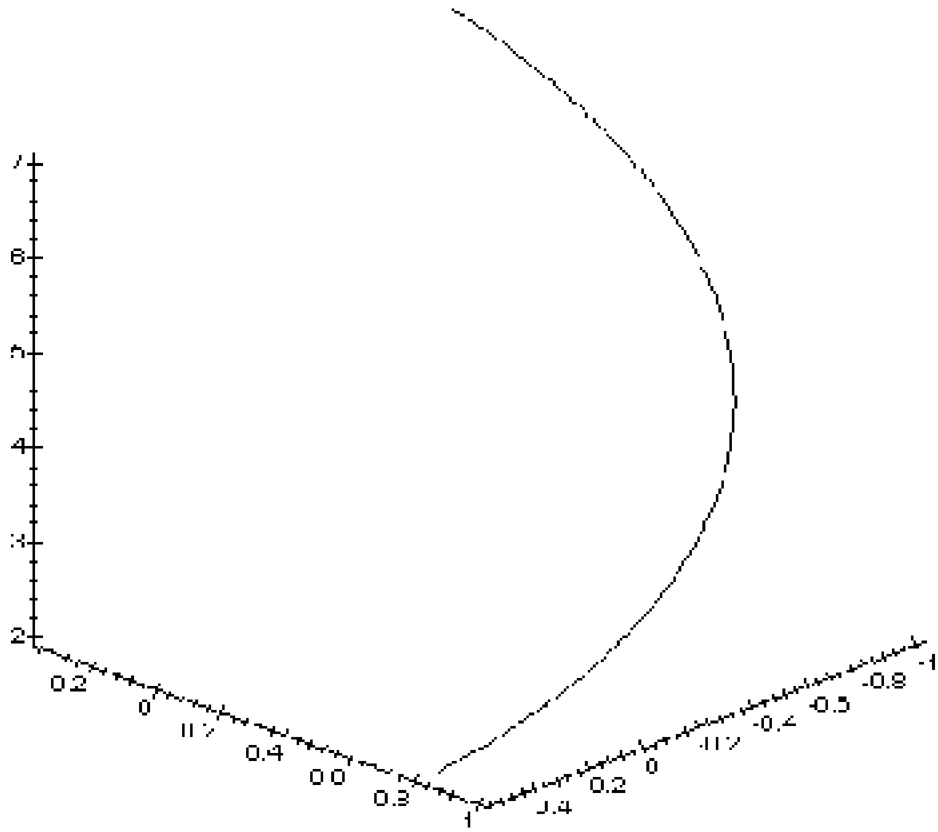

Figure 2: classical helix

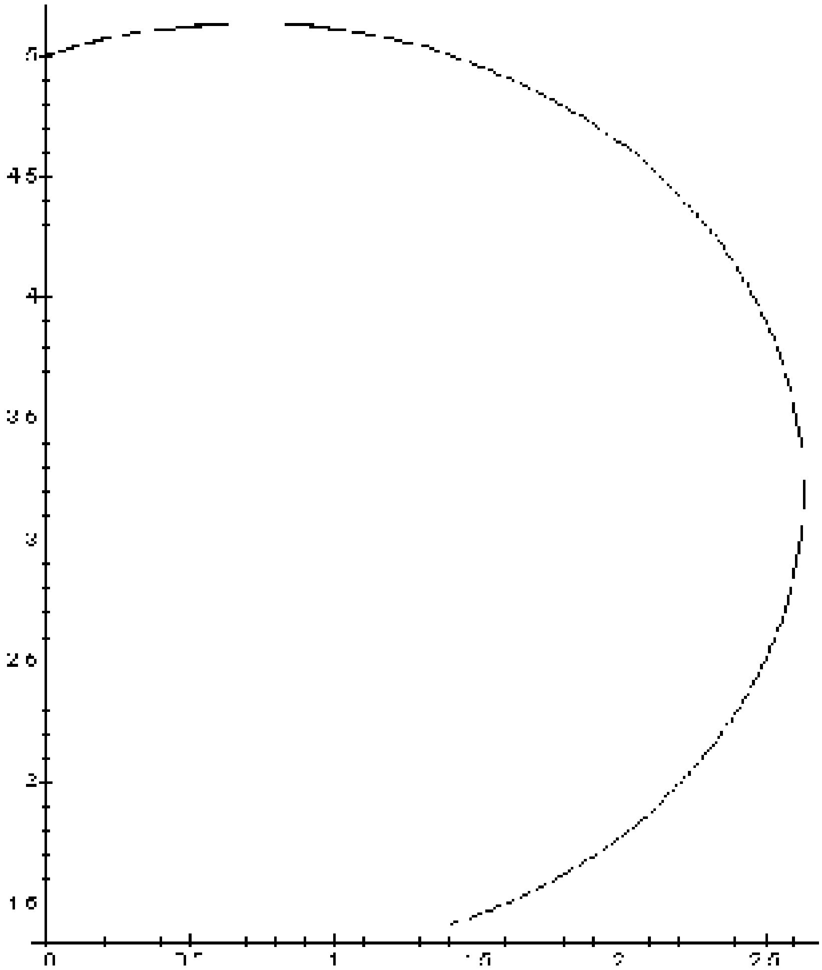

Figure 3: projection of the helix in the X-Y plane

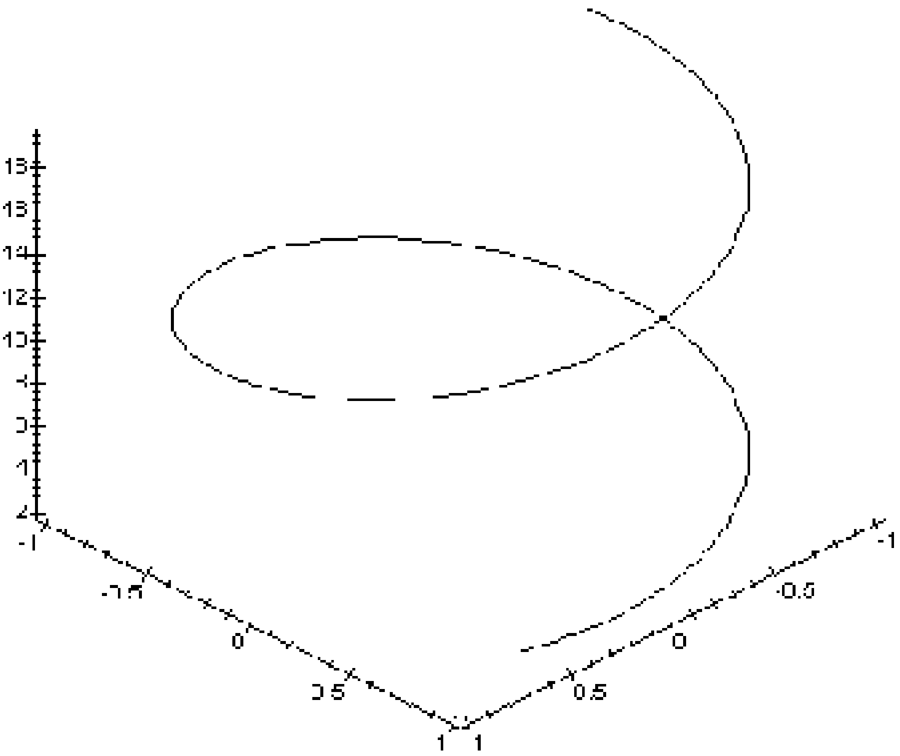

Figure 4: A generalized helix with vertical axis

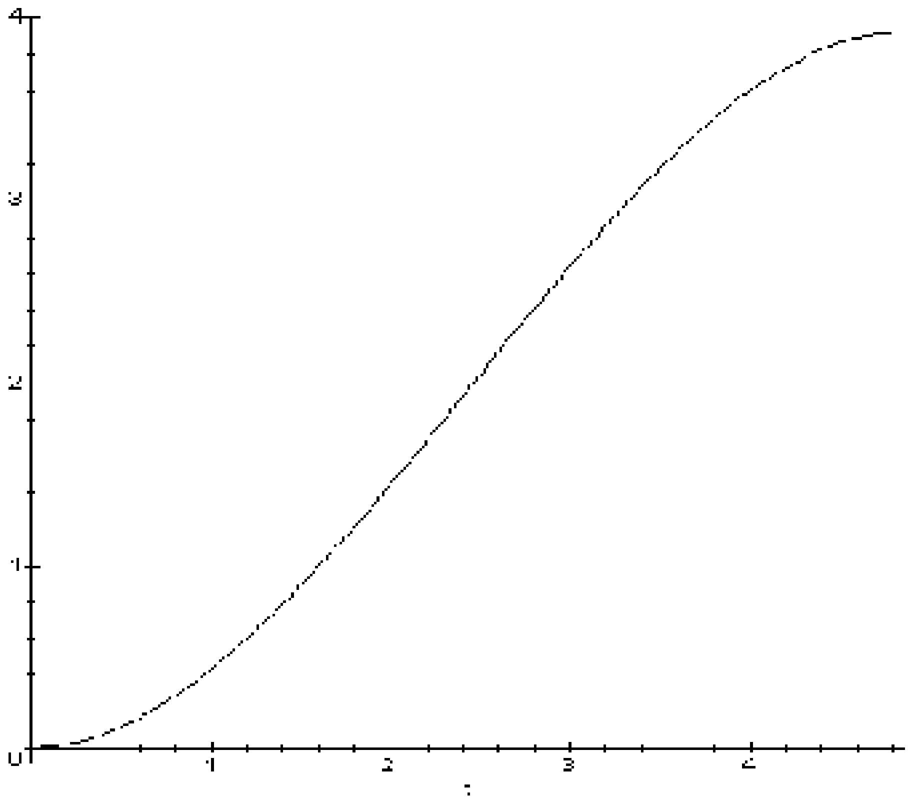

Figure 5: angle beta

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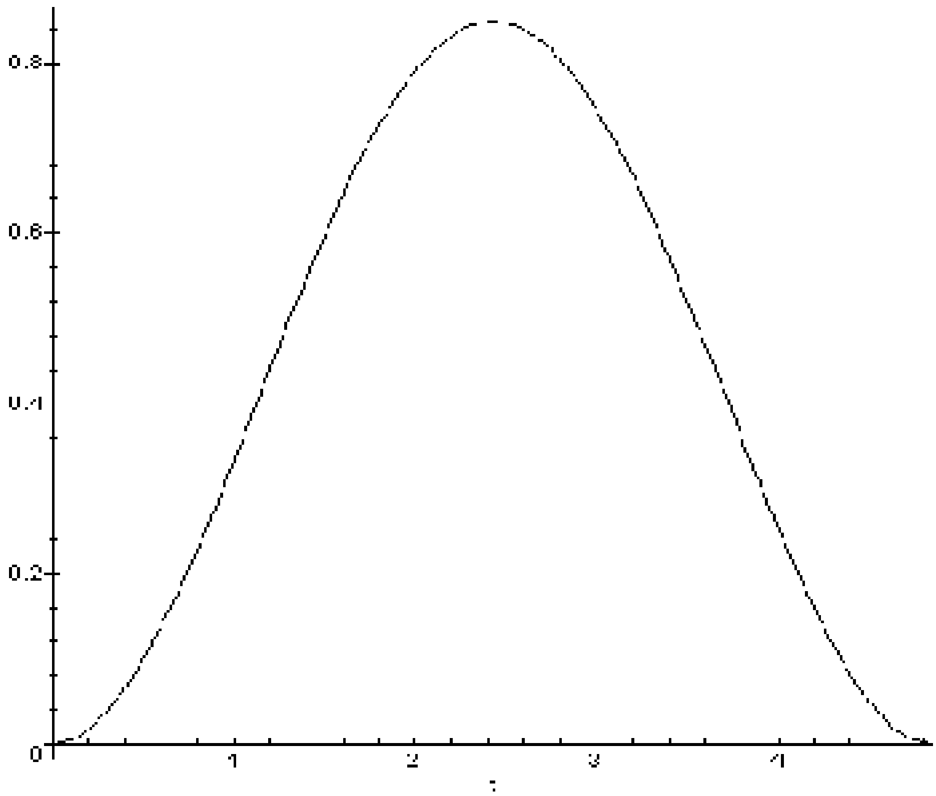

Figure 6: Curvature of the helix

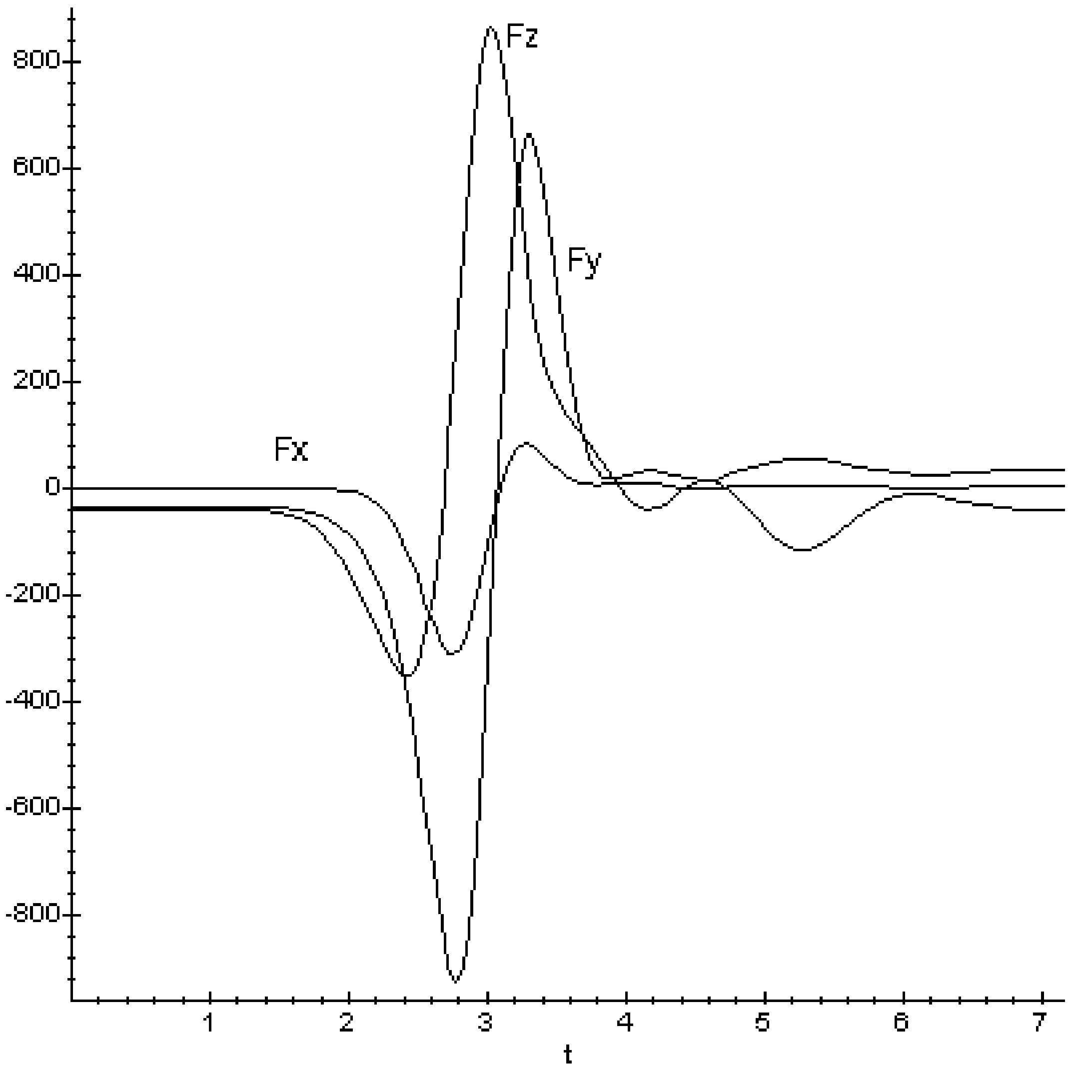

Figure 7: Forces in the X-Y-Z directions-kinematics constraints

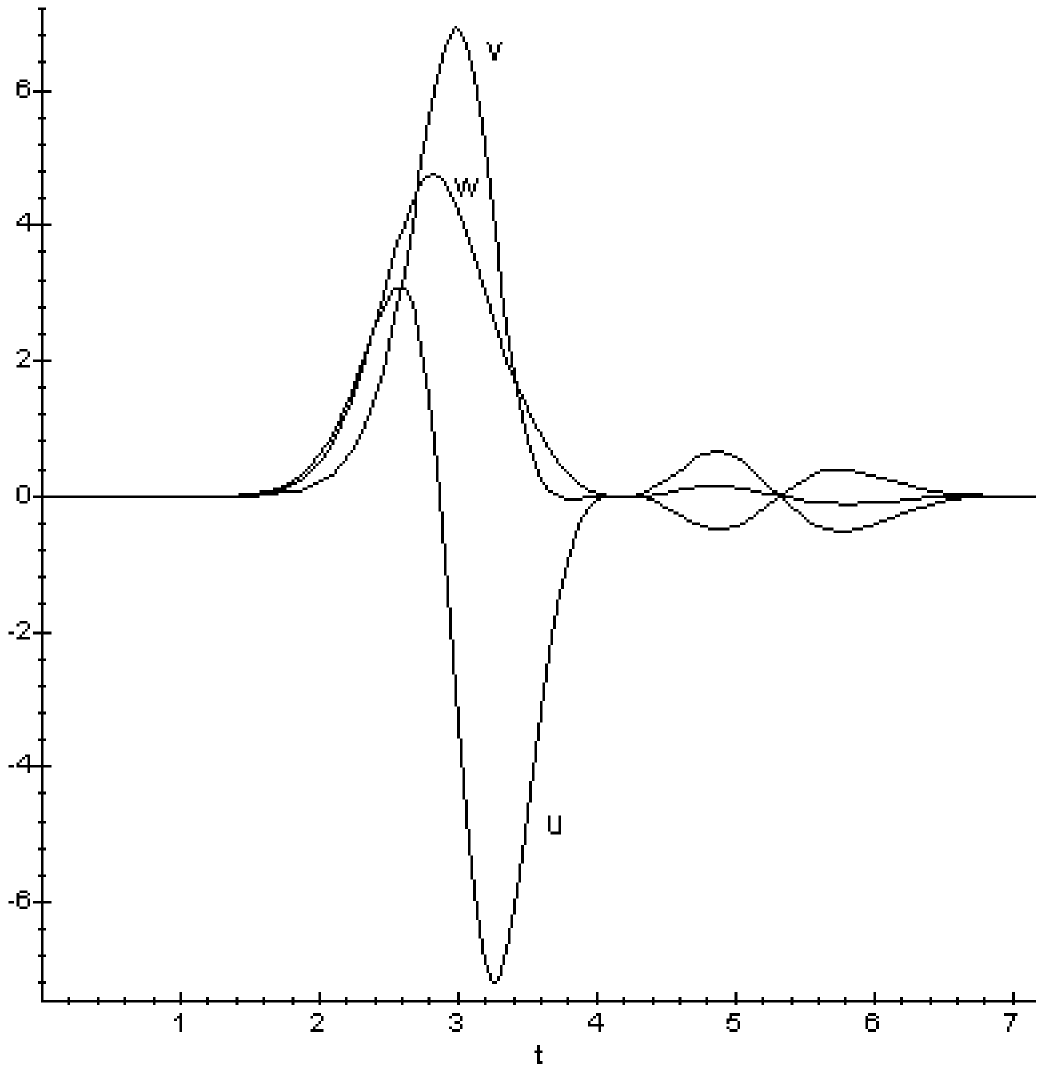

Figure 8: linear velocities-kinematics constraints

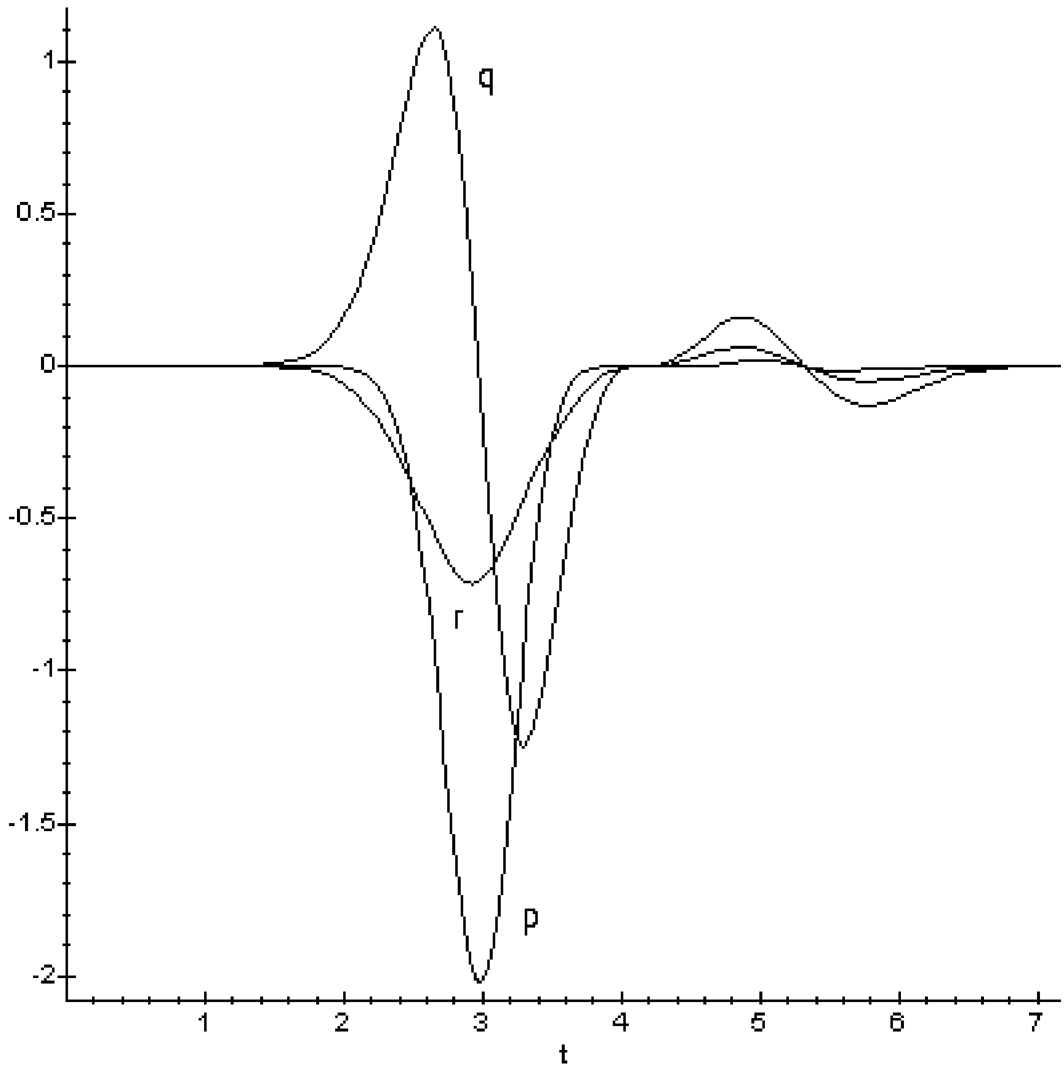

Figure 9: angular velocities-kinematics constraints

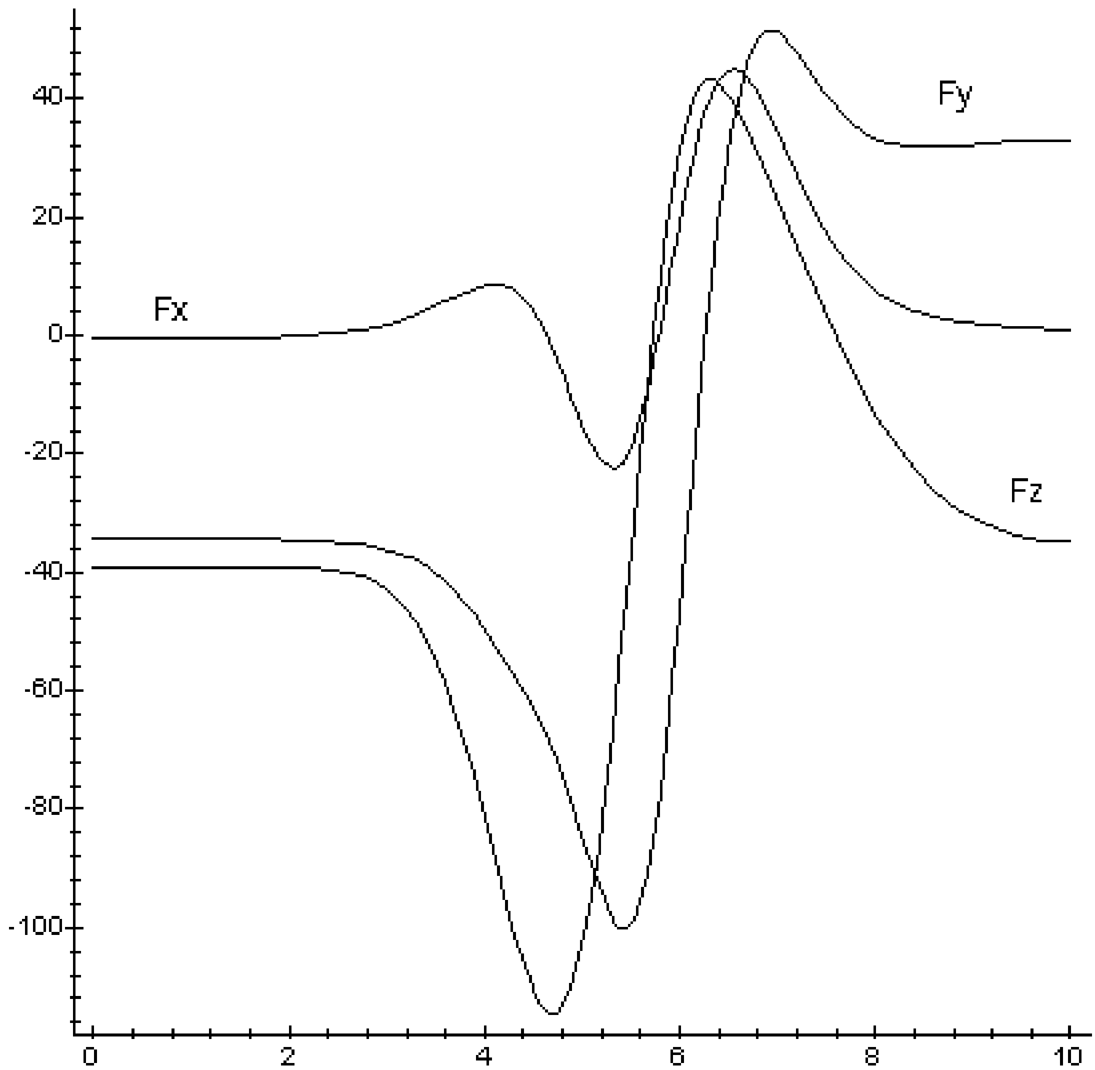

Figure 10: Forces in the X-Y-Z directions-dynamics constraints

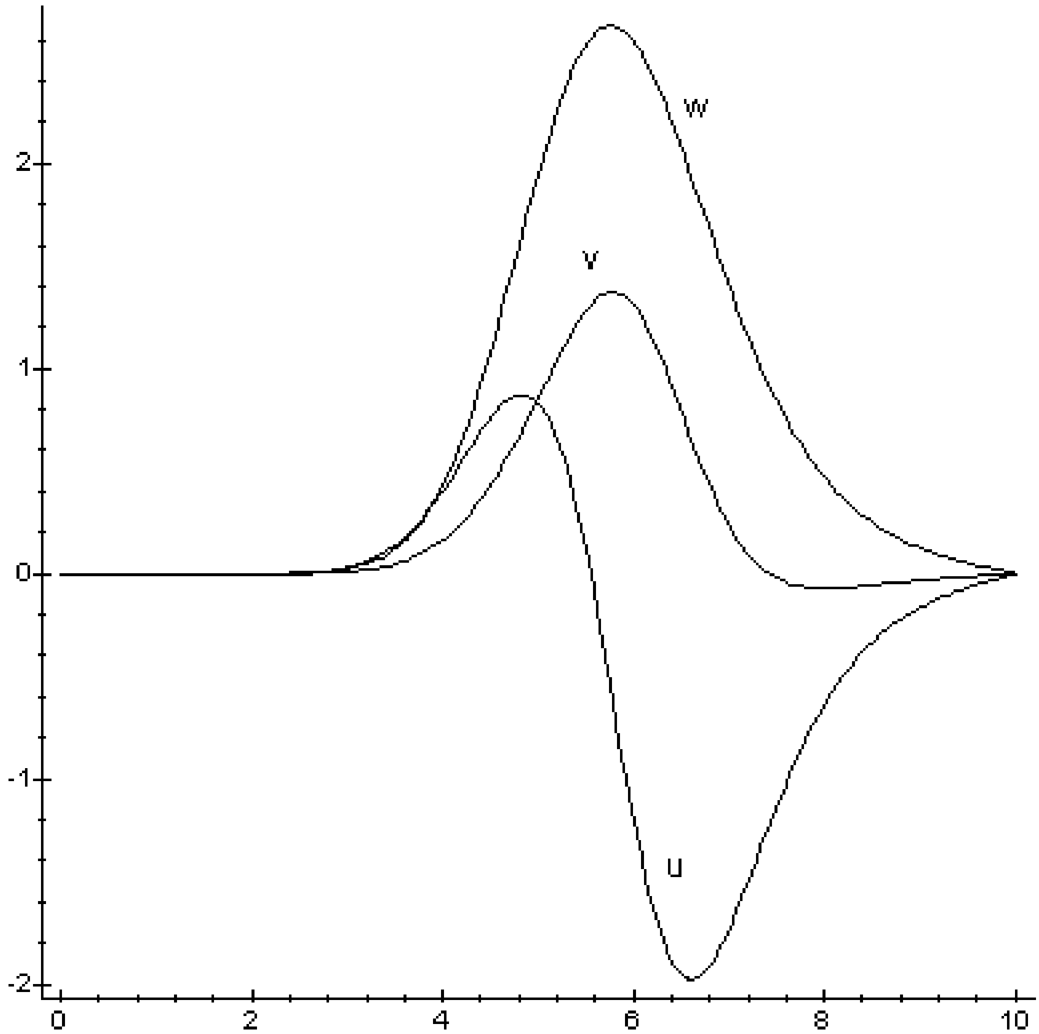

Figure 11: linear velocities-dynamics constraints

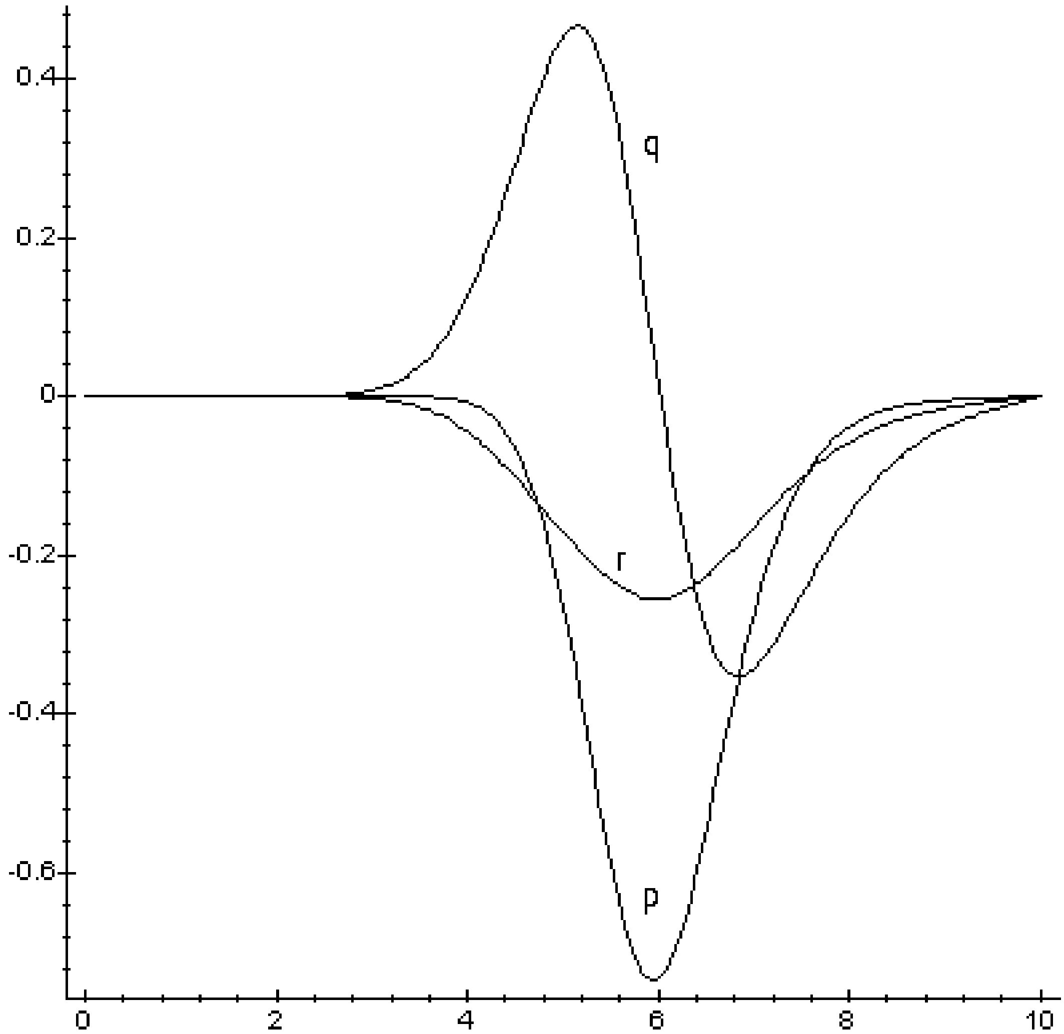

Figure 12: angular velocities-dynamics constraints