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Adaptive nonlinear output feedback for transient stabilization and voltage regulation of power generators with unknown parameters

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SUMMARY

This work presents a nonlinear adaptive output feedback excitation control, designed for a synchronous generator modelled by a standard third-order model on the basis of the physically available measurements of relative angular speed, active and reactive electric power and terminal voltage. The power angle, which is a crucial variable for the excitation control, as well as mechanical power and the impedance of the transmission line connecting the generator to an infinity bus, are not assumed to be available for feedback. The feedback control achieves transient stabilization and voltage regulation when faults occur to the turbines or the transmission lines, such that parameters (mechanical power and line impedance) may permanently take any (unknown) value. The controller recovers by adaptation the unknown values and simultaneously generates trajectories to be followed by the states, that converge to the new equilibrium point. Copyright © 2004 John Wiley & Sons, Ltd.

KEY WORDS: nonlinear control; adaptive control; power system stabilization; power generators

1. INTRODUCTION

Power system stabilization has been dealt with for many years by both control and power systems communities. For the latter, the goal is to have stable, reliable and robust electrical energy production and distribution. On the other hand, control system teams develop quite more complicated systems which may be difficult to implement. Our goal here is to present new control methods for power system stabilization, which are closer to physical considerations. These new control methods, mainly based on modern nonlinear techniques, may improve power

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systems stabilization since classical controllers found in most power plants have limitations in performance and in operation region.

On the other hand, the theoretical interest of these systems becomes evident as we remark that power generators are described by nonlinear equations with unknown time-varying parameters. There is no full state measurement, and they are underactuated systems. All these features make the problem quite difficult and interesting from a theoretical point of view. Its classical solution is presented in References [1, 2] using robust linear techniques that are widespread in most power plants. Modern linear robust and adaptive control techniques applied to this problem, may be seen in References [3–5]. Recently, feedback linearization [6–8] as well as nonlinear adaptive techniques [9, 10] were proposed to design stabilizing controllers with the purpose of enlarging the stability region of the operating condition.

The nonlinear feedback control algorithms so far proposed in the literature make use of power angle and mechanical power measurements, which are physically not available. These algorithms have also the difficulty of determining the faulted equilibrium value which is compatible with the required terminal voltage once the fault (mechanical or electrical failure) has occurred. This is our motivation to propose a nonlinear scheme based only on actually measured outputs. First, in Section 2, following the lines of our previous works [11–13], we make use of the standard third-order model used in Reference [10] (see References [2, 14]) to show that the terminal voltage, the relative angular speed and the active electric power (which are actually measurable and available for feedback) are state variables in the physical region of the state space. We then develop an adaptive feedback linearization of the system achieving exponential stability of the closed-loop system, as presented in Section 3. To do so, for a given set of unknown parameters, we recover, by adaptation, the new equilibrium point of the system and generate, on-line, a trajectory that drives the generator toward this point. This task becomes complicated as we have a nonlinear and nonlinearly parametrized system with unknown time-varying parameters, without full state measurement. Tracking in such systems is a difficult task, and has been recently studied for the SISO case in Reference [15]. We conclude the paper with simulations (Section 4) that show the good behaviour of the adaptive controller in the presence of transmission line and turbine faults.

2. DYNAMICAL MODEL

The power generator is represented by the standard model presented in Reference [2] (also used in References [7, 8, 10, 14]) that may be decomposed in a mechanical and an electrical parts. The advantage of such a model is that although being of low order, it expresses well the behaviour of large systems. This fact (model reduction) is well developed in Reference [16] where a mathematical approach leads to the same conclusions of standard physical simplifications. In practice, this may be seen as the Thevenin equivalent of a large network.

Let us first consider the simplified mechanical model expressed in per unit as

$$\begin{aligned} \dot{\delta} &= \omega \\ \dot{\omega} &= -\frac{D}{H}\omega + \frac{\omega_{\mathrm{s}}}{H}(P_{\mathrm{m}} - P_{\mathrm{e}}) \end{aligned} \tag{1}$$

where δ (rad) is the power angle of the generator relative to the angle of the infinite bus rotating at synchronous speed ω_s; ω (rad/s) is the angular speed of the generator relative to the synchronous speed ω_s i.e. $\omega = \omega_g - \omega_s$ with ω_g being the generator angular speed; $H(s)$ is the per unit inertia constant; D(p.u.) is the per unit damping constant; P_m(p.u.) is the per unit mechanical input power; P_e(p.u.) is the per unit active electric power delivered by the generator to the infinite bus. Note that the expression ω_s^2/ω_g is simplified as $\omega_s^2/\omega_g \simeq \omega_s$ in the right-hand side of (1). The active and reactive (Q(p.u.)) powers are given by

$$P_e = \frac{V_s E_q}{X_{ds}} \sin(\delta) \tag{2}$$

$$Q = \frac{V_s}{X_{ds}} E_q \cos(\delta) - \frac{V_s^2}{X_{ds}} \tag{3}$$

where E_q(p.u.) is the quadrature's EMF; V_s(p.u.) is the voltage at the infinite bus; X_{ds}(p.u.) $\triangleq X_T + \frac{1}{2}X_L + X_d$ is the total reactance which takes into account X_d(p.u.), the generator direct axis reactance, X_L(p.u.), the transmission line reactance, and X_T(p.u.), the reactance of the transformer. The quadrature EMF, E_q, and the transient quadrature EMF, E'_q, are related by

$$E_q = \frac{X_{ds}}{X'_{ds}} E'_q - \frac{X_d - X'_d}{X'_{ds}} V_s \cos(\delta) \tag{4}$$

while the dynamics of E'_q (representing the electrical part of the generator) are given by

$$\frac{dE'_q}{dt} = \frac{1}{T_{d0}} (K_c u_f - E_q) \tag{5}$$

in which X'_{ds}(p.u.) $\triangleq X_T + \frac{1}{2}X_L + X_d{}'$ with $X_d{}'$(p.u.) denoting the generator direct axis transient reactance; u_f(p.u.) is the input to the (SCR) amplifier of the generator; K_c is the gain of the excitation amplifier; $T_{d0}(s)$ is the direct axis short circuit time constant. Substituting (2) into (1) and (4) into (5), we obtain the state space model

$$\begin{aligned}
\dot{\delta} &= \omega \\
\dot{\omega} &= -\frac{D}{H}\omega + \frac{\omega_s}{H}\left(P_m - \frac{V_s}{X'_{ds}} E'_q \sin(\delta) + \frac{X_d - X'_d}{X_{ds} X'_{ds}} V_s^2 \sin(\delta)\cos(\delta)\right) \\
\dot{E'_q} &= \frac{1}{T_{d0}}\left(K_c u_f - \frac{X_{ds}}{X'_{ds}} E'_q + \frac{X_d - X'_d}{X'_{ds}} V_s \cos(\delta)\right)
\end{aligned} \tag{6}$$

in which (δ, ω, E'_q) is the state and u_f is the control input. Since P_e is measurable while E'_q is not, it is convenient to express the state space model using (δ, ω, P_e) as states which are equivalent states as long as the power angle δ remains in the open set $0 < \delta < \pi$.

In the following, we take into account the notation

$$T'_{d0} = \frac{X'_{ds}}{X_{ds}} T_{d0}$$

where T'_{d0} is the direct axis transient short circuit time constant. Differentiating (2) with respect to time, and using (1)–(5), we obtain

$$\dot{\delta} = \omega$$

$$\dot{\omega} = -\frac{D}{H}\omega - \frac{\omega_s}{H}(P_e - P_m)$$

$$\dot{P}_e = -\frac{1}{T'_{d0}}P_e + \frac{1}{T'_{d0}}\left\{\frac{V_s}{X_{ds}}\sin(\delta)\left[K_c u_f + T'_{d0}(X_d - X'_d)\frac{V_s}{X'_{ds}}\omega\sin(\delta)\right] + T'_{d0}P_e\omega\cot(\delta)\right\} \quad (7)$$

which is valid provided that $0 < \delta < \pi$. Note that when δ is near 0 or near π the effect of the input u_f on the overall dynamics is greatly reduced.

The generator terminal voltage is given by

$$V_t e^{j\varphi} = \frac{jX_s E_q e^{j(\pi/2+\delta)} + jX_d V_s e^{j\pi/2}}{jX_{ds}}$$

where

$$X_s = X_T + \frac{X_L}{2}$$

$$X_{ds} = X_d + X_s$$

so that its modulus is

$$V_t = \frac{1}{X_{ds}}(X_s^2 E_q^2 + V_s^2 X_d^2 + 2X_s X_d E_q V_s \cos(\delta))^{1/2}$$

or in the new state variables

$$V_t = \left(\frac{X_s^2 P_e^2}{V_s^2 \sin^2(\delta)} + \frac{X_d^2 V_s^2}{X_{ds}^2} + \frac{2X_s X_d}{X_{ds}} P_e \cot(\delta)\right)^{1/2} \quad (8)$$

which is the output of the system to be regulated to its reference value $V_{tr} = 1$(p.u.)

We must remark in this model that mechanical power, power angle and line impedance are not available for measurement. Actually, this is the main blocking point for nonlinear control of power generators.

We avoid this problem using the relation (see [11])

$$X_s = \frac{-QV_s^2 \pm \sqrt{Q^2 V_s^4 - (Q^2 + P_e^2)V_s^2(V_s^2 - V_t^2)}}{Q^2 + P_e^2} \quad (9)$$

to express the line impedance, and the relation

$$\delta = \operatorname{arccot}\left(\frac{V_s}{X_s P_e}\left(-\frac{X_d V_s}{X_{ds}} + \sqrt{V_t^2 - \frac{X_s^2}{V_s^2}P_e^2}\right)\right) \quad (10)$$

to express the power angle. With respect to the mechanical power, we will present an adaptive scheme to recover its value. Note that in Equation (9), we use X_s as the impedance of the line up to the point of the network where the voltage is equal to V_s. Errors in the infinity bus voltage will be expressed as a different value of line impedance, leading to an equivalent result for the controller.

One must also remark that (10) is a one-to-one function from δ to V_t (as V_t is positive). As a consequence, (V_t, ω, P_e), which are measurable and are available for feedback action, is an equivalent state for models (6) and (7).

3. NONLINEAR ADAPTIVE CONTROLLER

The operating conditions $(\delta_0, \omega_0, P_{e0})$ of the synchronous generator model (7) are given by

$$\begin{aligned}\omega_0 &= 0\\ P_{e0} &= P_m\\ -P_m + \frac{V_s}{X_{ds}} K_c u_f \sin(\delta) &= 0\end{aligned} \tag{11}$$

Note that while $\omega_0 = 0$, $P_{e0} = P_m$ are not affected by u_f, from the third equation above we see that there are two operating conditions δ_s, δ_u, $0<\delta_s<\pi/2$, $\pi/2<\delta_u<\pi$ for constant inputs $u_f > (P_m X_{ds})/(K_c V_s)$; $(\delta_s, 0, P_m)$ is an asymptotically stable equilibrium point while $(\delta_u, 0, P_m)$ is an unstable equilibrium point. The stable operating condition $(\delta_s, 0, P_m)$ and the corresponding excitation constant input

$$K_c u_{f0} = \frac{P_m X_{ds}}{V_s \sin(\delta_s)}$$

are chosen so that the modulus of the generator terminal voltage

$$V_t = \frac{1}{X_{ds}} (X_s^2 K_c^2 u_{f0}^2 + V_s^2 X_d^2 + 2 X_s X_d K_c u_{f0} V_s \cos(\delta_s))^{1/2}$$

is equal to the prescribed value V_{tr}.

The objective of the control system is to keep all states and outputs bounded and asymptotically bring outputs/states to their reference values. These objectives may be summarized as

$$\begin{array}{c} 0<\delta<180 \\ |\omega| \leqslant \omega_M < \infty, \\ |P_e| < \infty \end{array} \quad \lim_{t\to\infty} \begin{bmatrix} \omega \\ P_e \\ V_t \end{bmatrix} = \begin{bmatrix} 0 \\ P_m \\ V_{tr} \end{bmatrix}$$

where ω_M is a limit value for the angular velocity that is specified by the constructor.

One must remark that parameters may, and will, abruptly change in time. For instance, the parameter P_m may abruptly change to an unknown faulted value P_{mf} due to turbine failures, so that $(V_{tr}, 0, P_m)$ may not belong to the region of attraction of the faulted equilibrium point $(V_{tr}, 0, P_{mf})$. The state feedback control should be designed so that typical turbine failures do not cause instabilities and consequently loss of synchronism and inability to achieve voltage regulation.

A reduction from P_m to $(P_m)_f$ of the mechanical power generated by the turbine, changes the operating condition: the new operating condition $(\delta)_f$ is the solution of

$$-\frac{(P_m)_f}{P_m} + \frac{\sin(\delta)_f}{\sin(\delta_s)} = 0$$

and since $(P_m)_f$ is typically unknown, the corresponding new stable operating condition $(\delta_s)_f$ is also unknown. The control system must recover this new operation point, generate a trajectory towards it, and drive the system to this trajectory.

To develop the control, model (7) is rewritten as

$$
\begin{aligned}
\dot{\delta} &= \omega \\
\dot{\omega} &= -\frac{D}{H}\omega - \frac{\omega_s}{H}(P_e - \theta) \\
\dot{P}_e &= -\frac{1}{T'_{d0}}P_e + \frac{V_s}{X_{ds}T'_{d0}}\sin(\delta)K_c u_f + \frac{(X_d - X'_d)V_s^2}{X_{ds}X'_{ds}}\omega\sin^2(\delta) + P_e\omega\cot(\delta)
\end{aligned}
\tag{12}
$$

in which $\theta(t)$ is a possibly time-varying disturbance; the parameter θ is assumed to be unknown and to belong to the known compact set $[\theta_m, \theta_M]$ where the lower and upper bounds θ_m, θ_M are known.

Let $\delta_r(t)$ be a (at least) C^3 reference signal (toward the new equilibrium point) to be tracked. In order to build this trajectory (δ_r) toward the equilibrium value of the power angle (δ_s), we use Equation (13) where we replace V_t by its reference value V_{tr}; V_s is considered as 1(p.u.); X_s is the impedance of the line up to the point of the network where the voltage is equal to V_s, and is calculated by (9); X_d is a known constant and finally P_e is replaced by $\hat{P}_m$ that is the estimation of P_m. The resulting expression is

$$
\delta_r = \operatorname{arccot}\left(\left(\frac{V_s}{X_s\hat{P}_m}\right)\left(-\frac{V_s}{X_{ds}}X_d + \sqrt{V_{tr}^2 - \frac{X_s^2}{V_s^2}\hat{P}_m^2}\right)\right) \tag{13}
$$

As arccot(x) is a one-to-one smooth function, one may compute the correct δ_r for each set of arguments. Remark that as $\hat{P}_m$ goes to P_m, δ_r goes to δ_s.

In order to estimate P_m we define ($\hat{\omega}$ is an estimation of ω)

$$
\begin{aligned}
\tilde{P}_m &= P_m - \hat{P}_m \\
\tilde{\omega}_e &= (\omega - \hat{\omega})
\end{aligned}
$$

One must not confound this new defined $\tilde{\omega}_e$ with variable $\tilde{\omega}$ that we will define later. We may then write

$$
\begin{aligned}
\dot{\tilde{P}}_m &= -\dot{\hat{P}}_m = -\gamma_1\tilde{\omega}_e \\
\dot{\hat{\omega}} &= -\frac{D}{H}\hat{\omega} - \frac{\omega_s}{H}(P_e - \hat{P}_m)
\end{aligned}
$$

and then, using also the second equation of (7), we conclude that

$$
\dot{\tilde{\omega}}_e = -\frac{D}{H}\tilde{\omega}_e + \frac{\omega_s}{H}\tilde{P}_m
$$

or in a more concise form

$$
\begin{bmatrix} \dot{\tilde{P}}_m \\ \dot{\tilde{\omega}}_e \end{bmatrix} = \begin{bmatrix} 0 & -\gamma_1 \\ \frac{\omega_s}{H} & -\frac{D}{H} \end{bmatrix} \begin{bmatrix} \tilde{P}_m \\ \tilde{\omega}_e \end{bmatrix}
$$

which eigenvalues are

$$\lambda_i = \frac{1}{2}\frac{-D \pm \sqrt{D^2 - 4H\omega_s\gamma_1}}{H}$$

We may then see that a suitable choice of γ_1 will give an exponentially stable estimation. Actually, any $\gamma_1 > 0$ will meet this requirement, in particular a $0 < \gamma_1 \leqslant D^2/4H\omega_s$ that will give two negative real roots.

Next, we define

$$\tilde{\delta}(t) = \delta(t) - \delta_r(t)$$

where, taking the time derivative, we obtain

$$\dot{\tilde{\delta}} = \omega - \dot{\delta}_r(t)$$

As we want that the error system be a stable linear system, we state ω^* as the desired value for ω (taking $\lambda_1 > 0$):

$$\omega^* = -\lambda_1\tilde{\delta} + \dot{\delta}_r$$

and then we may define

$$\tilde{\omega} \triangleq \omega - \omega^* = \omega + \lambda_1\tilde{\delta} - \dot{\delta}_r$$

Taking the time derivatives of both equations leads to

$$\begin{aligned}
\dot{\tilde{\delta}} &= -\lambda_1\tilde{\delta} + \tilde{\omega} \\
\dot{\tilde{\omega}} &= -\frac{D}{H}\omega + \frac{\omega_s}{H}(\theta(t) - P_e) - \lambda_1^2\tilde{\delta} + \lambda_1\tilde{\omega} - \ddot{\delta}_r
\end{aligned} \tag{14}$$

Following the same technique, we define ($\lambda_2 > 0, k > 0$) the reference signal for P_e that linearizes our system:

$$P_e^* = \frac{H}{\omega_s}\left\{-\frac{D}{H}\omega - \lambda_1^2\tilde{\delta} + \lambda_1\tilde{\omega} - \ddot{\delta}_r + \lambda_2\tilde{\omega} + \tilde{\delta} + \frac{1}{4}k\left(\frac{\omega_s}{H}\right)^2\tilde{\omega}\right\} + \hat{\theta}$$

where $\hat{\theta}$ is an estimate of θ and

$$\tilde{P}_e = P_e - P_e^*$$

Rewriting the second equation of (14):

$$\dot{\tilde{\omega}} = \frac{\omega_s}{H}\tilde{\theta}(t) - \frac{\omega_s}{H}\tilde{P}_e - \lambda_2\tilde{\omega} - \tilde{\delta} - \frac{1}{4}k\left(\frac{\omega_s}{H}\right)^2\tilde{\omega}$$

and taking the derivative of P_e^*

$$\begin{aligned}
\dot{P}_e^* = \frac{H}{\omega_s}\Bigg\{&-\frac{D}{H}\left(\dot{\tilde{\omega}} - \lambda_1\dot{\tilde{\delta}} + \ddot{\delta}_r\right) \\
&+ \left(\lambda_1 + \lambda_2 + \frac{1}{4}k\left(\frac{\omega_s}{H}\right)^2\right)\left(-\frac{D}{H}\omega + \frac{\omega_s}{H}(\theta(t) - P_e) - \lambda_1^2\tilde{\delta} + \lambda_1\tilde{\omega} - \ddot{\delta}_r\right) \\
&+(1 - \lambda_1^2)(-\lambda_1\tilde{\delta} + \tilde{\omega})\Bigg\} + \dot{\hat{\theta}} - \frac{H}{\omega_s}\dddot{\delta}_r
\end{aligned}$$

Equation (12) can finally be rewritten as ($\tilde{\theta} = \theta - \hat{\theta}$)

$$
\begin{aligned}
\dot{\tilde{\delta}} &= -\lambda_1 \tilde{\delta} + \tilde{\omega} \\
\dot{\tilde{\omega}} &= -\tilde{\delta} - \lambda_2 \tilde{\omega} - \frac{\omega_s}{H} \tilde{P}_e - \frac{k}{4}\left(\frac{\omega_s}{H}\right)^2 \tilde{\omega} + \frac{\omega_s}{H} \tilde{\theta} \\
\dot{\tilde{P}}_e &= -\frac{1}{T'_{d0}} P_e + \frac{V_s}{X_{ds} T'_{d0}} \sin(\delta) K_c u_f + \frac{(X_d - X'_d) V_s^2}{X_{ds} X'_{ds}} \omega \sin^2(\delta) + P_e \omega \cot(\delta) \\
&\quad - \frac{H}{\omega_s} \Bigg\{ \left(-\lambda_1^2 + 1 + \lambda_1 \frac{D}{H}\right)(-\lambda_1 \tilde{\delta} + \tilde{\omega}) \\
&\quad + \left(-\frac{D}{H} + \lambda_1 + \lambda_2 + \frac{k}{4}\left(\frac{\omega_s}{H}\right)^2\right)\left(-\frac{D}{H}\omega - \lambda_1^2 \tilde{\delta} + \lambda_1 \tilde{\omega} - \frac{\omega_s}{H} P_e - \ddot{\delta}_r\right) \Bigg\} \\
&\quad - \left(-\frac{D}{H} + \lambda_1 + \lambda_2 + \frac{k}{4}\left(\frac{\omega_s}{H}\right)^2\right) \hat{\theta} - \dot{\hat{\theta}} \\
&\quad - \left(-\frac{D}{H} + \lambda_1 + \lambda_2 + \frac{k}{4}\left(\frac{\omega_s}{H}\right)^2\right) \tilde{\theta} + \frac{D}{\omega_s} \ddot{\delta}_r + \frac{H}{\omega_s} \dddot{\delta}_r
\end{aligned} \tag{15}
$$

We can see from Equation (15) that in order to compute our control signal we need the derivatives of δ_r. To do so, we must remember that

$$
\begin{aligned}
\dot{\delta}_r &= \frac{d\delta_r}{d\hat{P}_m} \frac{d\hat{P}_m}{dt} \\
\ddot{\delta}_r &= \frac{d^2\delta_r}{d\hat{P}_m^2} \frac{d\hat{P}_m}{dt} + \frac{d\delta_r}{d\hat{P}_m} \frac{d^2\hat{P}_m}{dt^2} \\
\dddot{\delta}_r &= \frac{d^3\delta_r}{d\hat{P}_m^3} \frac{d\hat{P}_m}{dt} + 2\frac{d^2\delta_r}{d\hat{P}_m^2} \frac{d^2\hat{P}_m}{dt^2} + \frac{d\delta_r}{d\hat{P}_m} \frac{d^3\hat{P}_m}{dt^3}
\end{aligned} \tag{16}
$$

These computations may be seen in Appendix A leading to

$$
\begin{aligned}
\dot{\delta}_r &= \gamma_1 \frac{d\delta_r}{d\hat{P}_m} \tilde{\omega}_e \\
\ddot{\delta}_r &= \gamma_1 \frac{d^2\delta_r}{d\hat{P}_m^2} \tilde{\omega}_e + \gamma_1 \frac{d\delta_r}{d\hat{P}_m} \frac{\omega_s}{H}\left(\hat{\theta} - \hat{P}_m\right) - \gamma_1 \frac{d\delta_r}{d\hat{P}_m} \frac{D}{H} \tilde{\omega}_e + \gamma_1 \frac{d\delta_r}{d\hat{P}_m} \frac{\omega_s}{H} \tilde{\theta} \\
\dddot{\delta}_r &= \gamma_1 \frac{d^3\delta_r}{d\hat{P}_m^3} \tilde{\omega}_e + 2\gamma_1 \frac{d^2\delta_r}{d\hat{P}_m^2} \frac{\omega_s}{H}\left(\hat{\theta} - \hat{P}_m\right) - 2\gamma_1 \frac{d^2\delta_r}{d\hat{P}_m^2} \frac{D}{H} \tilde{\omega}_e + 2\gamma_1 \frac{d^2\delta_r}{d\hat{P}_m^2} \frac{\omega_s}{H} \tilde{\theta} \\
&\quad + \left(\gamma_1 \frac{D^2}{H^2} - \gamma_1^2 \frac{\omega_s}{H}\right) \frac{d\delta_r}{d\hat{P}_m} \tilde{\omega}_e - \gamma_1 \frac{d\delta_r}{d\hat{P}_m} \frac{D\omega_s}{H^2} (\hat{\theta} - \hat{P}_m) - \gamma_1 \frac{d\delta_r}{d\hat{P}_m} \frac{D\omega_s}{H^2} \tilde{\theta}
\end{aligned}
$$

where $d\delta_r/d\hat{P}_m$, $d^2\delta_r/d\hat{P}_m^2$ and $d^3\delta_r/d\hat{P}_m^3$ are given by (A1), (A3) and (A5) in Appendix A.

Since some of the terms of $\ddot{\delta}_{\rm r}$ and $\dddot{\delta}_{\rm r}$ are not available for feedback, we define new variables $\ddot{\delta}_{{\rm r}u}$ and $\dddot{\delta}_{{\rm r}u}$ that will be used for our control law. These variables are defined such that

$$\begin{aligned}
\dot{\delta}_{\rm r} - \dot{\delta}_{{\rm r}u} &= 0 \\
\ddot{\delta}_{\rm r} - \ddot{\delta}_{{\rm r}u} &= \gamma_1 \frac{{\rm d}\delta_{\rm r}}{{\rm d}\hat{P}_{\rm m}} \frac{\omega_{\rm s}}{H} \tilde{\theta} \\
\dddot{\delta}_{\rm r} - \dddot{\delta}_{{\rm r}u} &= 2\gamma_1 \frac{{\rm d}^2\delta_{\rm r}}{{\rm d}\hat{P}_{\rm m}^2} \frac{\omega_{\rm s}}{H} \tilde{\theta} - \gamma_1 \frac{{\rm d}\delta_{\rm r}}{{\rm d}\hat{P}_{\rm m}} \frac{D\omega_{\rm s}}{H^2} \tilde{\theta}
\end{aligned}$$

Defining ($\lambda_3 > 0$), we may compute the control signal that will linearize the last equation of (15):

$$\begin{aligned}
u_f &= \frac{T'_{\rm d0} X_{\rm ds}}{V_{\rm s} K_c \sin(\delta)} \phi_0 \\
\phi_0 &= \frac{1}{T'_{\rm d0}} P_{\rm e} - \frac{(X_{\rm d} - X'_{\rm d})}{X_{\rm ds} X'_{\rm ds}} V_{\rm s}^2 \omega \sin^2(\delta) - P_{\rm e} \omega \cot(\delta) \\
&\quad + \frac{H}{\omega_{\rm s}} \Bigg\{ \left(-\lambda_1^2 + 1 + \lambda_1 \frac{D}{H} \right) (-\lambda_1 \tilde{\delta} + \tilde{\omega}) \\
&\quad + \left(-\frac{D}{H} + \lambda_1 + \lambda_2 + \frac{k}{4} \left(\frac{\omega_{\rm s}}{H} \right)^2 \right) \left(-\frac{D}{H} \omega - \lambda_1^2 \tilde{\delta} + \lambda_1 \tilde{\omega} - \frac{\omega_{\rm s}}{H} P_{\rm e} - \ddot{\delta}_{{\rm r}u} \right) \Bigg\} \\
&\quad + \left(-\frac{D}{H} + \lambda_1 + \lambda_2 + \frac{k}{4} \left(\frac{\omega_{\rm s}}{H} \right)^2 \right) \hat{\theta} + \dot{\hat{\theta}} \\
&\quad - \frac{k}{4} \left(-\frac{D}{H} + \lambda_1 + \lambda_2 + \frac{k}{4} \left(\frac{\omega_{\rm s}}{H} \right)^2 \right)^2 \tilde{P}_{\rm e} - \lambda_3 \tilde{P}_{\rm e} + \frac{\omega_{\rm s}}{H} \tilde{\omega} - \frac{D}{\omega_{\rm s}} \ddot{\delta}_{{\rm r}u} - \frac{H}{\omega_{\rm s}} \dddot{\delta}_{{\rm r}u}
\end{aligned}$$

Remark here the use of $\delta_{{\rm r}u}$ as the feedback available variable. Now, defining the new constant

$$c_1 \triangleq \left(-\frac{D}{H} + \lambda_1 + \lambda_2 + \frac{k}{4} \left(\frac{\omega_{\rm s}}{H} \right)^2 \right)$$

we may rewrite the previous equations as

$$\begin{aligned}
u_f &= \frac{T'_{\rm d0} X_{\rm ds}}{V_{\rm s} K_c \sin(\delta)} \phi_0 \\
\phi_0 &= \frac{1}{T'_{\rm d0}} P_{\rm e} - \frac{(X_{\rm d} - X'_{\rm d})}{X_{\rm ds} X'_{\rm ds}} V_{\rm s}^2 \omega \sin^2(\delta) - P_{\rm e} \omega \cot(\delta) + c_1 \hat{\theta} + \dot{\hat{\theta}} \\
&\quad + \frac{H}{\omega_{\rm s}} \left\{ \left(-\lambda_1^2 + 1 + \lambda_1 \frac{D}{H} \right) (-\lambda_1 \tilde{\delta} + \tilde{\omega}) + c_1 \left(-\frac{D}{H} \omega - \lambda_1^2 \tilde{\delta} + \lambda_1 \tilde{\omega} - \frac{\omega_{\rm s}}{H} P_{\rm e} - \ddot{\delta}_{{\rm r}u} \right) \right\} \\
&\quad - \frac{k}{4} \left(c_1 - \gamma_1 c_1 \frac{{\rm d}\delta_{\rm r}}{{\rm d}\hat{P}_{\rm m}} - 2\gamma_1 \frac{{\rm d}^2\delta_{\rm r}}{{\rm d}\hat{P}_{\rm m}^2} \right)^2 \tilde{P}_{\rm e} - \lambda_3 \tilde{P}_{\rm e} + \frac{\omega_{\rm s}}{H} \tilde{\omega} - \frac{D}{\omega_{\rm s}} \ddot{\delta}_{{\rm r}u} - \frac{H}{\omega_{\rm s}} \dddot{\delta}_{{\rm r}u}
\end{aligned} \tag{17}$$

and

$$
\begin{aligned}
\dot{\tilde{P}}_{\mathrm{e}} = & -\frac{1}{T'_{\mathrm{d0}}} P_{\mathrm{e}} + \frac{(X_{\mathrm{d}} - X'_{\mathrm{d}})V_{\mathrm{s}}^2}{X_{\mathrm{ds}} X'_{\mathrm{ds}}} \omega \sin^2(\delta) + P_{\mathrm{e}} \omega \cot(\delta) \\
& - \frac{H}{\omega_{\mathrm{s}}} \left\{ \left(-\lambda_1^2 + 1 + \lambda_1 \frac{D}{H} \right)(-\lambda_1 \tilde{\delta} + \tilde{\omega}) + c_1 \left(-\frac{D}{H} \omega - \lambda_1^2 \tilde{\delta} + \lambda_1 \tilde{\omega} - \frac{\omega_{\mathrm{s}}}{H} P_{\mathrm{e}} - \ddot{\delta}_{\mathrm{r}} \right) \right\} \\
& - c_1 \hat{\theta} - \dot{\hat{\theta}} - c_1 \tilde{\theta} + \frac{D}{\omega_{\mathrm{s}}} \ddot{\delta}_{\mathrm{r}} + \frac{H}{\omega_{\mathrm{s}}} \dddot{\delta}_{\mathrm{r}} + \frac{V_{\mathrm{s}}}{X_{\mathrm{ds}} T'_{\mathrm{d0}}} \sin(\delta) K_c \frac{T'_{\mathrm{d0}} X_{\mathrm{ds}}}{V_{\mathrm{s}} K_c \sin(\delta)} \phi_0
\end{aligned} \tag{18}
$$

Substituting (17) in (18) one will find

$$
\begin{aligned}
\dot{\tilde{P}}_{\mathrm{e}} = & -\frac{1}{T'_{\mathrm{d0}}} P_{\mathrm{e}} + \frac{(X_{\mathrm{d}} - X'_{\mathrm{d}})V_{\mathrm{s}}^2}{X_{\mathrm{ds}} X'_{\mathrm{ds}}} \omega \sin^2(\delta) + P_{\mathrm{e}} \omega \cot(\delta) \\
& - \frac{H}{\omega_{\mathrm{s}}} \left\{ \left(-\lambda_1^2 + 1 + \lambda_1 \frac{D}{H} \right)(-\lambda_1 \tilde{\delta} + \tilde{\omega}) + c_1 \left(-\frac{D}{H} \omega - \lambda_1^2 \tilde{\delta} + \lambda_1 \tilde{\omega} - \frac{\omega_{\mathrm{s}}}{H} P_{\mathrm{e}} - \ddot{\delta}_{\mathrm{r}} \right) \right\} \\
& - c_1 \hat{\theta} - \dot{\hat{\theta}} - c_1 \tilde{\theta} + \frac{D}{\omega_{\mathrm{s}}} \ddot{\delta}_{\mathrm{r}} + \frac{H}{\omega_{\mathrm{s}}} \dddot{\delta}_{\mathrm{r}} \\
& + \frac{1}{T'_{\mathrm{d0}}} P_{\mathrm{e}} - \frac{(X_{\mathrm{d}} - X'_{\mathrm{d}})}{X'_{\mathrm{ds}} X_{\mathrm{ds}}} V_{\mathrm{s}}^2 \omega \sin^2(\delta) - P_{\mathrm{e}} \omega \cot(\delta) \\
& + \frac{H}{\omega_{\mathrm{s}}} \left\{ \left(-\lambda_1^2 + 1 + \lambda_1 \frac{D}{H} \right)(-\lambda_1 \tilde{\delta} + \tilde{\omega}) + c_1 \left(-\frac{D}{H} \omega - \lambda_1^2 \tilde{\delta} + \lambda_1 \tilde{\omega} - \frac{\omega_{\mathrm{s}}}{H} P_{\mathrm{e}} - \ddot{\delta}_{\mathrm{r}u} \right) \right\} \\
& + c_1 \hat{\theta} + \dot{\hat{\theta}} - \frac{k}{4} \left(c_1 - \gamma_1 c_1 \frac{\mathrm{d}\delta_{\mathrm{r}}}{\mathrm{d}\hat{P}_{\mathrm{m}}} - 2\gamma_1 \frac{\mathrm{d}^2 \delta_{\mathrm{r}}}{\mathrm{d}\hat{P}_{\mathrm{m}}^2} \right)^2 \tilde{P}_{\mathrm{e}} - \lambda_3 \tilde{P}_{\mathrm{e}} + \frac{\omega_{\mathrm{s}}}{H} \tilde{\omega} - \frac{D}{\omega_{\mathrm{s}}} \ddot{\delta}_{\mathrm{r}u} - \frac{H}{\omega_{\mathrm{s}}} \dddot{\delta}_{\mathrm{r}u}
\end{aligned}
$$

that may be rewritten as

$$
\dot{\tilde{P}}_{\mathrm{e}} = -\frac{k}{4} \left(c_1 - \gamma_1 c_1 \frac{\mathrm{d}\delta_{\mathrm{r}}}{\mathrm{d}\hat{P}_{\mathrm{m}}} - 2\gamma_1 \frac{\mathrm{d}^2 \delta_{\mathrm{r}}}{\mathrm{d}\hat{P}_{\mathrm{m}}^2} \right)^2 \tilde{P}_{\mathrm{e}} - \lambda_3 \tilde{P}_{\mathrm{e}} + \frac{\omega_{\mathrm{s}}}{H} \tilde{\omega} - \left(c_1 - \gamma_1 c_1 \frac{\mathrm{d}\delta_{\mathrm{r}}}{\mathrm{d}\hat{P}_{\mathrm{m}}} - 2\gamma_1 \frac{\mathrm{d}^2 \delta_{\mathrm{r}}}{\mathrm{d}\hat{P}_{\mathrm{m}}^2} \right) \tilde{\theta}
$$

Then, the closed-loop system becomes

$$
\begin{aligned}
\dot{\tilde{\delta}} &= -\lambda_1 \tilde{\delta} + \tilde{\omega} \\
\dot{\tilde{\omega}} &= -\tilde{\delta} - \lambda_2 \tilde{\omega} - \frac{\omega_{\mathrm{s}}}{H} \tilde{P}_{\mathrm{e}} - \frac{k}{4} \left(\frac{\omega_{\mathrm{s}}}{H} \right)^2 \tilde{\omega} + \frac{\omega_{\mathrm{s}}}{H} \tilde{\theta} \\
\dot{\tilde{P}}_{\mathrm{e}} &= -\frac{k}{4} \left(c_1 - \gamma_1 c_1 \frac{\mathrm{d}\delta_{\mathrm{r}}}{\mathrm{d}\hat{P}_{\mathrm{m}}} - 2\gamma_1 \frac{\mathrm{d}^2 \delta_{\mathrm{r}}}{\mathrm{d}\hat{P}_{\mathrm{m}}^2} \right)^2 \tilde{P}_{\mathrm{e}} \\
& \quad - \lambda_3 \tilde{P}_{\mathrm{e}} + \frac{\omega_{\mathrm{s}}}{H} \tilde{\omega} - \left(c_1 - \gamma_1 c_1 \frac{\mathrm{d}\delta_{\mathrm{r}}}{\mathrm{d}\hat{P}_{\mathrm{m}}} - 2\gamma_1 \frac{\mathrm{d}^2 \delta_{\mathrm{r}}}{\mathrm{d}\hat{P}_{\mathrm{m}}^2} \right) \tilde{\theta}
\end{aligned} \tag{19}
$$

The adaptation law is (γ is a positive adaptation gain)

$$\dot{\hat{\theta}} = \gamma \,\mathrm{Proj}\left(\left(-\tilde{P}_{\mathrm{e}}\left(c_1 - \gamma_1 c_1 \frac{\mathrm{d}\delta_{\mathrm{r}}}{\mathrm{d}\hat{P}_{\mathrm{m}}} - 2\gamma_1 \frac{\mathrm{d}^2\delta_{\mathrm{r}}}{\mathrm{d}\hat{P}_{\mathrm{m}}^2}\right) + \tilde{\omega}\frac{\omega_{\mathrm{s}}}{H}\right), \hat{\theta}\right) \tag{20}$$

where $\mathrm{Proj}(y, \hat{\theta})$ is the smooth projection algorithm introduced in Reference [17]

$$\begin{aligned}
&\mathrm{Proj}(y, \hat{\theta}) = y \quad \text{if } p(\hat{\theta}) \leqslant 0\\
&\mathrm{Proj}(y, \hat{\theta}) = y \quad \text{if } p(\hat{\theta}) \geqslant 0 \text{ and } \langle \mathrm{grad}\, p(\hat{\theta}), y\rangle \leqslant 0\\
&\mathrm{Proj}(y, \hat{\theta}) = [1 - p(\hat{\theta})|\mathrm{grad}\, p(\hat{\theta})|] \quad \text{otherwise}
\end{aligned} \tag{21}$$

with

$$p(\theta) = \frac{(\theta - (\theta_M + \theta_{\mathrm{m}})/2)^2 - ((\theta_M - \theta_{\mathrm{m}})/2)}{\varepsilon^2 + 2\varepsilon((\theta_M - \theta_{\mathrm{m}})/2)}$$

for ε an arbitrary positive constant, which guarantees in particular that

(i) $\theta_{\mathrm{m}} - \varepsilon \leqslant \hat{\theta}(t) \leqslant \theta_M + \varepsilon$
(ii) $|\mathrm{Proj}(y, \hat{\theta})| \leqslant |y|$
(iii) $(\theta - \hat{\theta})\,\mathrm{Proj}(y, \hat{\theta}) \geqslant (\theta - \hat{\theta})y$

We may remark that if $\tilde{P}_{\mathrm{e}}$ and $\tilde{\omega}$ were the errors from the state to an equilibrium point, the adaptation law would be equivalent to a gradient approach. But this analysis, globally, is not true since these two errors signals are not the state errors. Only in a small region around the equilibrium point this would be valid.

To compute this adaptation law, let us consider the function

$$W = \tfrac{1}{2}(\tilde{\delta}^2 + \tilde{\omega}^2 + \tilde{P}_{\mathrm{e}}^2) \tag{22}$$

whose time derivative, according to (19), is

$$\begin{aligned}
\dot{W} = &- \lambda_1\tilde{\delta}^2 - \lambda_2\tilde{\omega}^2 - \lambda_3\tilde{P}_{\mathrm{e}}^2 + \tilde{\omega}\frac{\omega_{\mathrm{s}}}{H}\tilde{\theta} - \frac{k}{4}\left(\frac{\omega_{\mathrm{s}}}{H}\right)^2\tilde{\omega}^2\\
&- \frac{k}{4}\left(c_1 - \gamma_1 c_1 \frac{\mathrm{d}\delta_{\mathrm{r}}}{\mathrm{d}\hat{P}_{\mathrm{m}}} - 2\gamma_1\frac{\mathrm{d}^2\delta_{\mathrm{r}}}{\mathrm{d}\hat{P}_{\mathrm{m}}^2}\right)^2\tilde{P}_{\mathrm{e}}^2 - \left(c_1 - \gamma_1 c_1\frac{\mathrm{d}\delta_{\mathrm{r}}}{\mathrm{d}\hat{P}_{\mathrm{m}}} - 2\gamma_1\frac{\mathrm{d}^2\delta_{\mathrm{r}}}{\mathrm{d}\hat{P}_{\mathrm{m}}^2}\right)\tilde{\theta}\tilde{P}_{\mathrm{e}}
\end{aligned}$$

Completing the squares, we obtain the inequality

$$\dot{W} \leqslant -\lambda_1\tilde{\delta}^2 - \lambda_2\tilde{\omega}^2 - \lambda_3\tilde{P}_{\mathrm{e}}^2 + \frac{2}{k}\tilde{\theta}^2 \tag{23}$$

which guarantees arbitrary $\mathscr{L}_\infty$ robustness from the parameter error $\tilde{\theta}$ to the tracking errors $\tilde{\delta}, \tilde{\omega}, \tilde{P}_{\mathrm{e}}$ (see Reference [18, Section 5.4]).

The projection algorithms (21) guarantee that $\tilde{\theta}$ is bounded, and, by virtue of (22) and (23), that $\tilde{\delta}$, $\tilde{\omega}$ and $\tilde{P}_{\mathrm{e}}$ are bounded. Therefore, $\dot{\hat{\theta}}$ is bounded. Integrating (23), we have for every $t \geqslant t_0 \geqslant 0$

$$-\int_{t_0}^{t}(\lambda_1\tilde{\delta}^2 + \lambda_2\tilde{\omega}^2 + \lambda_3\tilde{P}_{\mathrm{e}}^2)\,\mathrm{d}\tau + \frac{2}{k}\int_{t_0}^{t}\tilde{\theta}^2\,\mathrm{d}\tau \geqslant W(t) - W(t_0)$$

Since $W(t) \geqslant 0$ and, by virtue of the projection algorithm (21),

$$\tilde{\theta}(t) \leqslant \theta_M - \theta_\mathrm{m} + \varepsilon$$

it follows that

$$\int_{t_0}^{t} (\lambda_1 \tilde{\delta}^2 + \lambda_2 \tilde{\omega}^2 + \lambda_3 \tilde{P}_\mathrm{e}^2)\, \mathrm{d}\tau \leqslant W(t_0) + \frac{2}{k}(\theta_M - \theta_\mathrm{m} + \varepsilon)^2 (t - t_0)$$

which, if $W(t_0) = 0$ (i.e. t_0 is a time before the occurrence of the fault), implies arbitrary $\mathscr{L}_2$ attenuation (by a factor k) of the errors $\tilde{\delta}$, $\tilde{\omega}$ and $\tilde{P}_\mathrm{e}$ caused by the fault. To analyse the asymptotic behaviour of the adaptive control, we consider the function

$$V = \frac{1}{2}(\tilde{\delta}^2 + \tilde{\omega}^2 + \tilde{P}_\mathrm{e}^2) + \frac{1}{2}\frac{1}{\gamma}\tilde{\theta}^2$$

Its time derivative is

$$\begin{aligned}\dot{V} = &- \lambda_1 \tilde{\delta}^2 - \lambda_2 \tilde{\omega}^2 - \lambda_3 \tilde{P}_\mathrm{e}^2 + \tilde{\omega}\frac{\omega_\mathrm{s}}{H}\tilde{\theta} - \frac{k}{4}\left(\frac{\omega_\mathrm{s}}{H}\right)^2 \tilde{\omega}^2 + \frac{1}{\gamma}\tilde{\theta}\dot{\tilde{\theta}} \\ &- \left(c_1 - \gamma_1 c_1 \frac{\mathrm{d}\delta_\mathrm{r}}{\mathrm{d}\hat{P}_\mathrm{m}} - 2\gamma_1 \frac{\mathrm{d}^2\delta_\mathrm{r}}{\mathrm{d}\hat{P}_\mathrm{m}^2}\right)\tilde{\theta}\tilde{P}_\mathrm{e} - \frac{k}{4}\left(c_1 - \gamma_1 c_1 \frac{\mathrm{d}\delta_\mathrm{r}}{\mathrm{d}\hat{P}_\mathrm{m}} - 2\gamma_1 \frac{\mathrm{d}^2\delta_\mathrm{r}}{\mathrm{d}\hat{P}_\mathrm{m}^2}\right)^2 \tilde{P}_\mathrm{e}^2\end{aligned}$$

Then, using the adaptation law, we may find (remember that $(\dot{\tilde{\theta}} = -\dot{\hat{\theta}})$):

$$\dot{V} = -\lambda_1 \tilde{\delta}^2 - \lambda_2 \tilde{\omega}^2 - \lambda_3 \tilde{P}_\mathrm{e}^2 - \frac{k}{4}\left(\frac{\omega_\mathrm{s}}{H}\right)^2 \tilde{\omega}^2 - \frac{k}{4}\left(c_1 - \gamma_1 c_1 \frac{\mathrm{d}\delta_\mathrm{r}}{\mathrm{d}\hat{P}_\mathrm{m}} - 2\gamma_1 \frac{\mathrm{d}^2\delta_\mathrm{r}}{\mathrm{d}\hat{P}_\mathrm{m}^2}\right)^2 \tilde{P}_\mathrm{e}^2$$

The projection estimation algorithm (21) is designed so that the time derivative of V satisfies

$$\dot{V} \leqslant -\lambda_1 \tilde{\delta}^2 - \lambda_2 \tilde{\omega}^2 - \lambda_3 \tilde{P}_\mathrm{e}^2 \tag{24}$$

Integrating (24), we have

$$\lim_{t\to\infty} \int_{t_0}^{t} (\lambda_1 \tilde{\delta}^2 + \lambda_2 \tilde{\omega}^2 + \lambda_3 \tilde{P}_\mathrm{e}^2)\, \mathrm{d}\tau \leqslant V(0) - V(\infty) < \infty$$

From the boundedness of $\dot{\tilde{\delta}}$, $\dot{\tilde{\omega}}$ and $\dot{\tilde{P}}_\mathrm{e}$ and Barbalat's Lemma (see References [19–21]) it follows that

$$\lim_{t\to\infty} \left\| \begin{bmatrix} \tilde{\delta}(t) \\ \tilde{\omega}(t) \\ \tilde{P}_\mathrm{e}(t) \end{bmatrix} \right\| = 0$$

We may now rewrite the closed-loop system following the normal form:

$$\dot{\tilde{x}} = A\tilde{x} + \Omega^\mathrm{T}\tilde{\theta}$$

$$\dot{\tilde{\theta}} = -\Lambda\Omega\tilde{x}$$

which leads to

$$\dot{\tilde{x}} = \begin{bmatrix} -\lambda_1 & 1 & 0 \\ -1 & -\left(\lambda_2 + \frac{k}{4}\left(\frac{\omega_s}{H}\right)^2\right) & -\frac{\omega_s}{H} \\ 0 & \frac{\omega_s}{H} & -\left(\lambda_3 + \frac{k}{4}\left(-\frac{D}{H} + \lambda_1 + \lambda_2 + \frac{k}{4}\left(\frac{\omega_s}{H}\right)^2\right)^2\right) \end{bmatrix} \tilde{x}$$

$$+ \begin{bmatrix} 0 \\ \frac{\omega_s}{H} \\ -\left(-\frac{D}{H} + \lambda_1 + \lambda_2 + \frac{k}{4}\left(\frac{\omega_s}{H}\right)^2\right) \end{bmatrix} \tilde{\theta}$$

$$\dot{\tilde{\theta}} = -\gamma \left[0 \quad \frac{\omega_s}{H} \quad \left(\frac{D}{H} - \lambda_1 - \lambda_2 - \frac{k}{4}\left(\frac{\omega_s}{H}\right)^2\right) \right] \tilde{x} \tag{25}$$

And then computing (for a constant c_2):

$$\Omega\Omega^{\mathrm{T}} = \frac{\omega_s^2}{H^2} + \left(-\frac{D}{H} + \lambda_1 + \lambda_2 + \frac{k}{4}\left(\frac{\omega_s}{H}\right)^2\right)^2 \triangleq c_2 > 0$$

we then may show by persistency of excitation (see References [19–21]) that $\tilde{x}$ and $\tilde{\theta}$ will be globally exponentially stable, and then all error signals go exponentially to zero, for all (at least) C^3 $\delta_r(\hat{P}_m, x)$.

It is important to remark a very interesting feature of the proposed controller: all states go exponentially to the faulted equilibrium point that is completely unknown. Actually, all states go exponentially to trajectories that go themselves exponentially to the unknown equilibrium point. We must also remark that both convergencies are simultaneous. To detail this feature, note that δ will converge to the trajectory δ_r but, since δ_r is a one-to-one smooth function of $\hat{P}_m$, it will converge exponentially to the correct equilibrium value δ_s as $\hat{P}_m$ converges exponentially to P_m. This means that the reference trajectory (δ_r) will converge exponentially to the unknown equilibrium point (δ_s), and this convergence will be simultaneous to the convergence of the power angle (δ) to the trajectory (δ_r), what implies that $\lim_{t\to\infty}(\delta - \delta_s) = 0$ exponentially. We must remark that the same happens to the other states (ω and P_e). They converge to their reference trajectories $[\omega^*, P_e^*]$, and these trajectories converge to the faulted (unknown for P_e) equilibrium points of ω and P_e as $\tilde{\omega}(t)$ and $\tilde{P}_e(t)$ converge to zero.

Remark

We must observe that there are two adapted values for the mechanical power. The reason is that even if both results finally recover the same value, they are not used for the same purpose, neither as equivalent variables. Note that $\hat{P}_m$ is the estimation of the unknown parameter P_m, replacing it in the process of building the trajectories. It was designed purposely as an estimator and its behaviour can be defined as desired, such that it can respect the restrictions imposed for our trajectories, mainly with respect to being at least C^3. Furthermore, its time derivatives, that are needed for the controller, are available. As a consequence, $\hat{P}_m$ is very well behaved, going smoothly to the correct value of P_m.

On the other hand, $\hat{\theta}$ was designed as the control adaptation. Even if it finally recovers the correct value of P_m (faster than $\hat{P}_m$ in some cases), it is not as well behaved, nor its time derivatives are available. As a control variable, it was expected to be swift. That is what assures the awareness of the control signal, being able to act very fast to assure the stability of the power generator.

4. SIMULATION RESULTS

In this section we present simulations of the proposed controller, using the following data:

$$\omega_s = 314.159 \text{ rad/s}, \quad D = 5 \text{ p.u.}, \quad H = 8 \text{ s}$$

$$T_{d0} = 6.9 \text{ s}, \quad K_c = 1, \quad X_d = 1.863 \text{ p.u.}$$

$$X'_d = 0.257 \text{ p.u.}, \quad X_T = 0.127 \text{ p.u.}, \quad X_L = 0.4853 \text{ p.u.}$$

The operating point is $\delta_s = 72^\circ$, $P_m = 0.9$ p.u., $\omega_0 = 0$ to which corresponds $V_t = 1$ p.u., with $V_s = 1$ p.u.

The goal of the first simulation was to verify the effect of a severe fault on the turbine. It was considered a fast reduction of the mechanical input power, and the simulation was performed according to the following sequence:

1. The system is in pre-faulted state.
2. At $t = 0.5$ s the mechanical input power begins to decrease.
3. At $t = 5.5$ s the mechanical input power is 50% of the initial value.

The simulations were carried out using as control parameters:

$$\lambda_1 = 2, \quad \lambda_2 = 10, \quad \lambda_3 = 100$$

$$\gamma = 0.1, \quad k = 0.01, \quad \gamma_1 = 3D^2/4H\omega_s$$

Figure 1(a) shows that the trajectory for the power angle (δ_r) goes smoothly to its final value (δ_s), and that δ matches it almost perfectly, being driven to its faulted unknown equilibrium point.

In Figure 1(b) we see that the rotor velocity is correctly and smoothly driven to its equilibrium value, as well as the electrical power, driven to its trajectory that finally recovers the unknown equilibrium value as we may remark in Figure 1(c).

Figure 2(1a) shows how the output voltage drops during the fault, and goes to its correct value when the system is driven to the correct equilibrium point. If the estimation were not correct, there would be a steady state error.

One can see in Figure 2(1b) that the control signal is very smooth and is kept inside the prescribed bounds.

We may see in Figure 2(2a) the adapted value, $\hat{\theta}$, (dashed line) of the mechanical power (full line). It is accurate and swift, such that the correct value is adapted almost at once. We may remark that it recovers the correct value faster than the estimator does, as we may see in Figure 2(2b) where it is plotted the estimated $\hat{P}_m$ (dashed line) and the mechanical power (full line). On

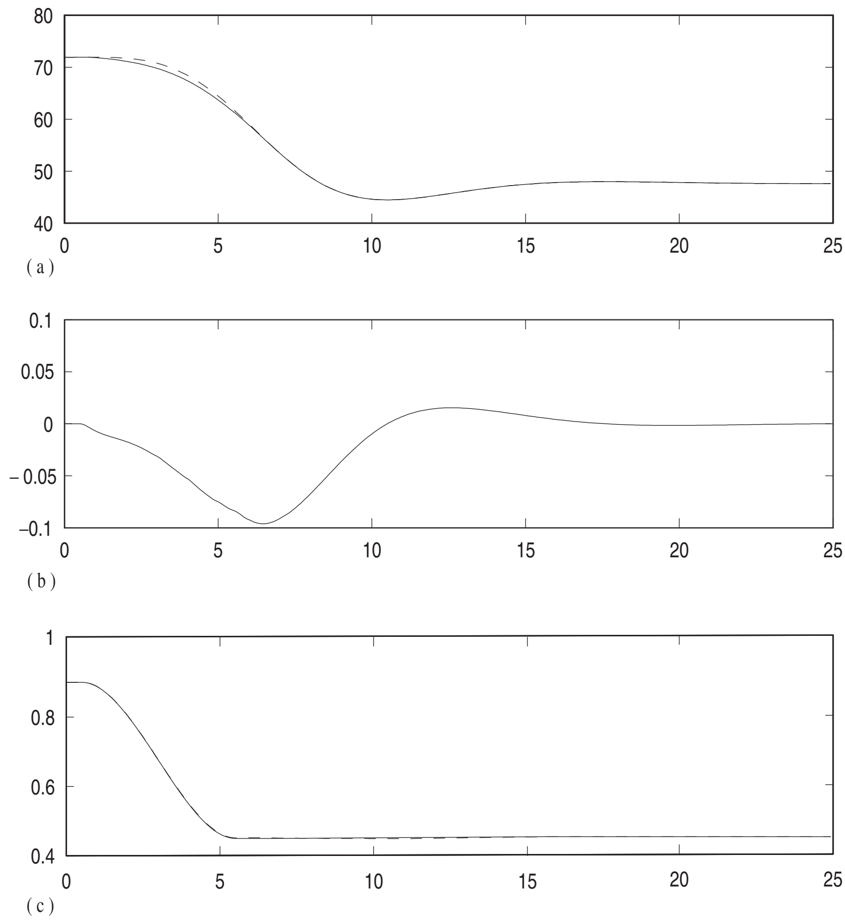

Figure 1. (a) δ (-), δ_r (- -), (b) ω, (c) P_e (- -), P_m (-).

the other hand, the estimated value $\hat{P}_m$ is very smooth, respecting the restrictions on the derivatives imposed for our tracked trajectory.

Note that during all time, the errors are very small. They can be made even smaller by increasing the parameter k. The choice of parameters is mainly based on the limitation of the control signal, as well as the desired bounds for states and outputs.

We present now the effect of faults on the transmission line. It was considered a large increment of line impedance, followed by an almost as large reduction. This is equivalent to the lost of part of the transmission lines, followed by a partial recover. Simulations were carried out following the sequence:

1. The system is in pre-faulted state.
2. At $t = 1$ s part of the power lines falls. This is reflected by an increment of line impedance in 33%. Note that the change is instantaneous.

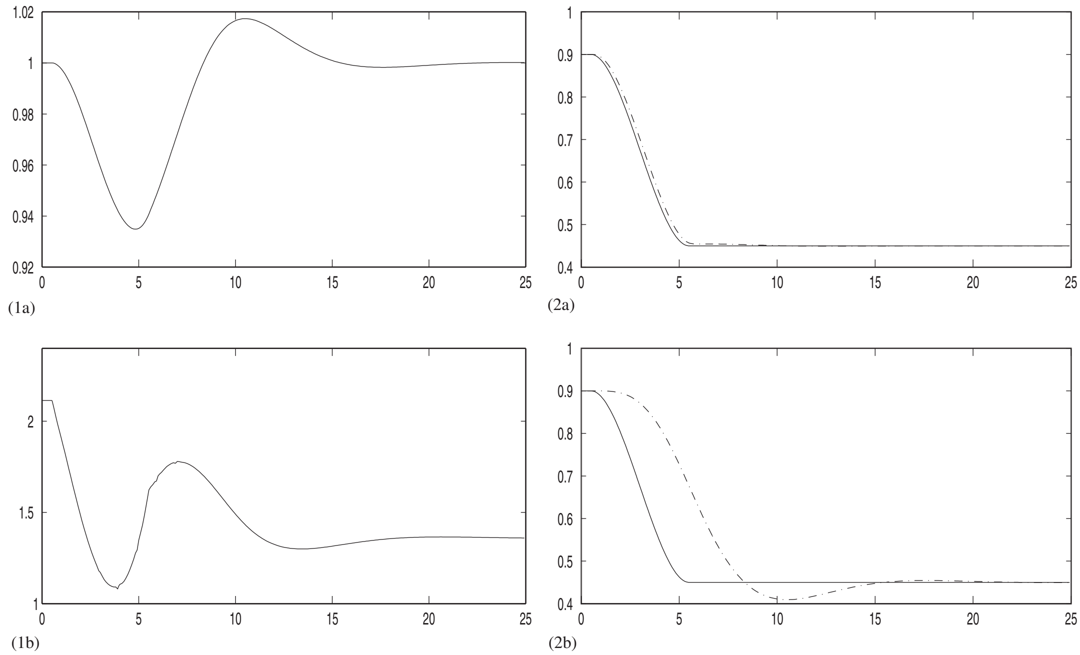

Figure 2. (1a) V_t, (1b) Control signal, (2a) $\hat{\theta}$ (-.) P_m (-), (2b) $\hat{P}_m$ (-.), P_m (-).

3. At $t = 5$ s part of the lines are recovered. This is seen as a reduction of 25% of the initial value of the line impedance.

The control parameters used for the simulations in this case are

$$\lambda_1 = 2, \quad \lambda_2 = 10, \quad \lambda_3 = 100$$
$$\gamma = 0.1, \quad k = 0.01, \quad \gamma_1 = 3D^2/4H\omega_s$$

Figure 3(a) shows that the trajectory (dashed line) for the power angle (δ_r) goes smoothly to its final value (δ_s), and that δ (full line) is able to track this trajectory, such that it is driven to its faulted equilibrium point.

In Figure 3(b) and 3(c) we may see the other two states, the rotor velocity and the electrical power, being disturbed by the faults and then driven to their correct values by the controller. The same is verified in Figure 4(1a) for the output voltage.

One can see in Figure 4(1b) that the control signal is very fast, acting at once to keep the stability of our system. It is able to keep all signals inside the prescribed bounds, and to drive them to their correct values. Contrariwise the previous simulation where, as a mechanical fault, the perturbation was quite slow, here we see an electrical fault, then a much faster one, asking for a sharp response from the controller.

We may observe in Figure 4(2a) the control adaptation variable $\hat{\theta}$ (dashed line) and the mechanical power (full line). In Figure 4(2b), it is presented the estimation $\hat{P}_m$ (dashed line) of the mechanical power P_m (full line). One may then remark that both

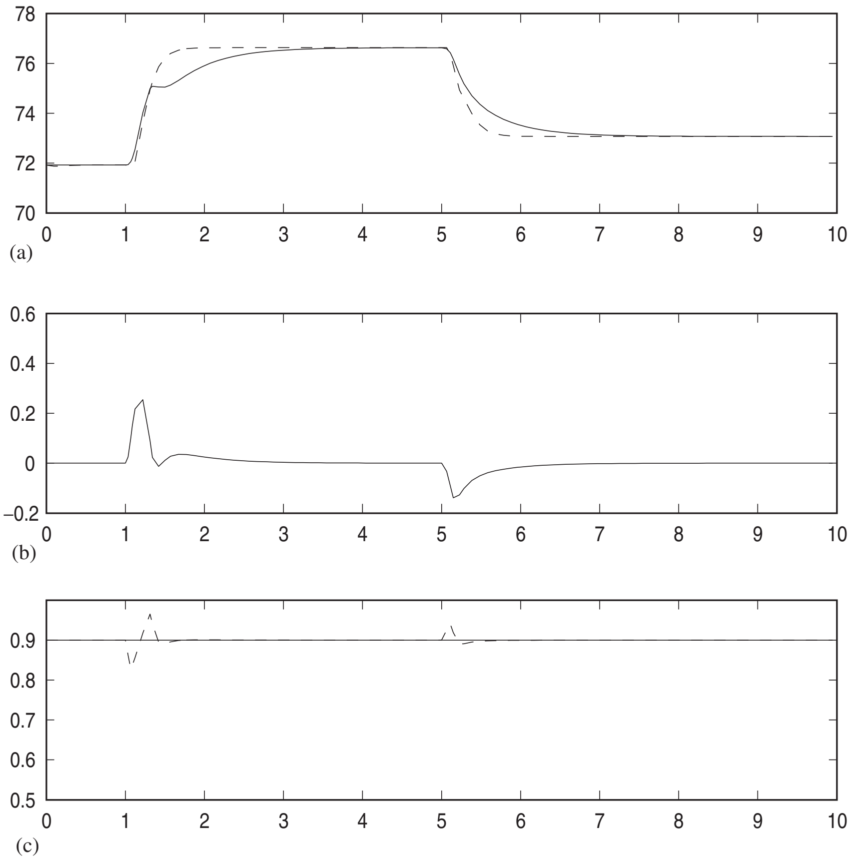

Figure 3. (a) δ (-), δ_r (- -), (b) ω, (c) P_e (- -), P_m (-).

variables recover the same final value, but while $\hat{P}_m$ keeps unchanged, $\hat{\theta}$ changes in time. This shows the difference between $\hat{\theta}$, as control variable, and $\hat{P}_m$ as estimated value.

Finally, in Figure 4(2c), one may see that the correct value for the transmission line impedance is computed by our technique. The value is recovered very fast, such that the system may be driven to its correct equilibrium point. This computation is filtered in order to respect physical limitations on the control signal magnitude.

5. CONCLUSION

In this paper, we have treated the problem of exponentially stabilizing a power generator using available output measurement. The proposed controller may be implemented in practice since only actually measured outputs are used for feedback. Usually, nonlinear controllers found in literature need the mechanical power, the transmission line impedance and the power angle, which make them not implementable. On the other hand, the linear controllers, usually implemented in power plants, do not assure a large stability region, and are not able to stand large perturbations.

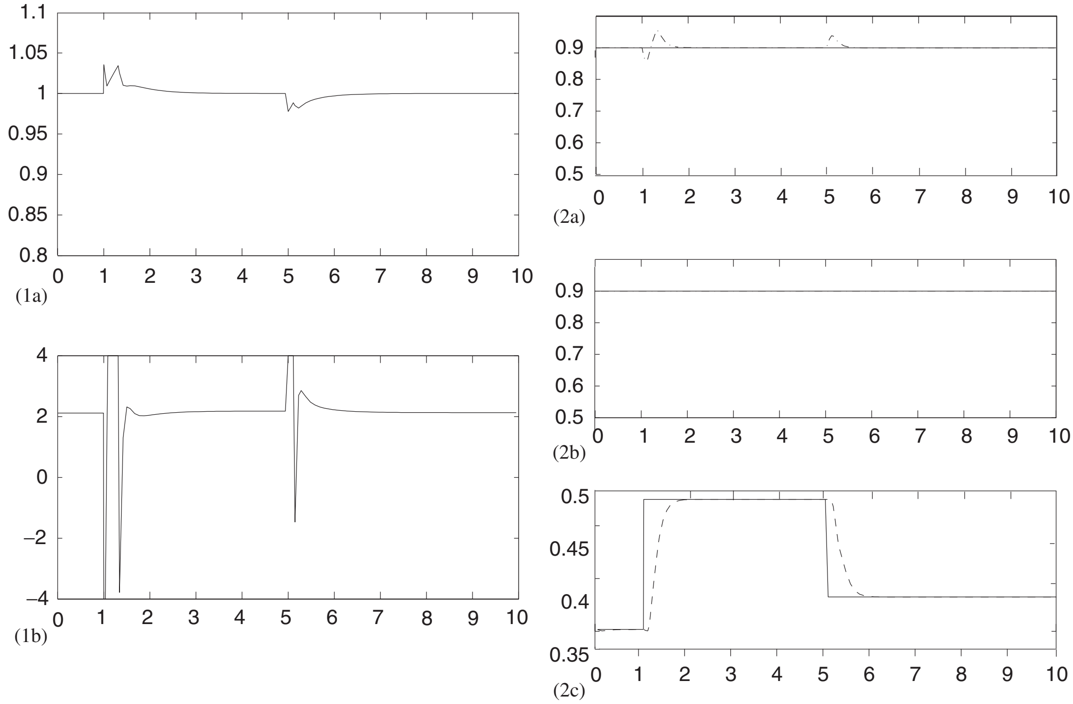

Figure 4. (1a) V_t, (1b) Control signal, (2a) $\hat{\theta}$ (-.), P_m (-), (2b) $\hat{P}_m$ (- -), P_m (-), (2c) X_s (-), Computed X_s (- -).

To design the proposed controller, we have first developed techniques to compute the unknown parameters such that the equilibrium point may be recovered after a fault or parameters changes. We then design trajectories (one for each state) toward this new point that are tracked by the states, driven by the controller. This is achieved by an adaptive output feedback linearization scheme designed using backstepping techniques, that also assures boundedness of all signals. The convergence of the trajectories to the equilibrium point is simultaneous to the convergence of the states toward the trajectories and the generation of these trajectories is made on-line by an exponentially stable adaptive estimator that recovers the mechanical power value.

Finally we present simulation results that corroborate our claims. They show the good behaviour of all states, outputs and control signal even in the presence of severe faults on turbine and on transmission line.

As further developments, our main goal is to extend these results to the multi-machine case. Actually, the single-machine study is a step toward the more general (and in practice the most important) case of multiple interconnected generators undergoing interzone oscillations. Since, in general, power plants are located very far from each others, centralized controllers that need information from each machine in the system are not realistic. The scheme proposed in this paper could be a starting point in the design of decentralized controllers.

APPENDIX A

In the following, we will compute the terms $d\delta_r/d\hat{P}_m$, $d^2\delta_r/d\hat{P}_m^2$, $d^3\delta_r/d\hat{P}_m^3$, $d\hat{P}_m/dt$, $d^2\hat{P}_m/dt^2$ and $d^3\hat{P}_m/dt^3$ in order to build Equation (16).

For the sake of simplicity, we first define

$$a = \frac{V_s}{X_s}$$
$$b = \frac{V_s X_d}{X_{ds}}$$

such that we may rewrite (13) as

$$\delta_r = \operatorname{arccot}\left(\frac{a\left(-b + \sqrt{V_{tr}^2 - \hat{P}_m^2/a^2}\right)}{\hat{P}_m}\right)$$

Using

$$\frac{d\,\operatorname{arccot}(x)}{dx} = -\frac{1}{1+x^2}$$

we compute

$$\frac{d\delta_r}{d\hat{P}_m} = -\frac{-\frac{a\left(-b+\sqrt{V_{tr}^2-\hat{P}_m^2/a^2}\right)}{\hat{P}_m^2} - \frac{1}{a\sqrt{V_{tr}^2-\hat{P}_m^2/a^2}}}{1+\frac{a^2\left(-b+\sqrt{V_{tr}^2-\hat{P}_m^2/a^2}\right)^2}{\hat{P}_m^2}}$$
$$\triangleq (N_1 + N_2) * \mathrm{Den} \tag{A1}$$

where

$$\mathrm{Den} = -\frac{1}{1 + a^2\left(-b + \sqrt{V_{tr}^2 - \hat{P}_m^2/a^2}\right)^2/\hat{P}_m^2}$$
$$N_1 = -a\left(-b + \sqrt{V_{tr}^2 - \hat{P}_m^2/a^2}\right)/\hat{P}_m^2$$
$$N_2 = -1/a\sqrt{V_{tr}^2 - \hat{P}_m^2/a^2}$$

Now, recalling that

$$\frac{d^2\delta_r}{d\hat{P}_m^2} = \left(\frac{d\,\mathrm{Den}}{d\hat{P}_m} * (N_1 + N_2) + \mathrm{Den} * \left(\frac{dN_1}{d\hat{P}_m} + \frac{dN_2}{d\hat{P}_m}\right)\right) \tag{A2}$$

we first compute

$$\frac{dN_1}{d\hat{P}_m} = 2\frac{a\left(-b+\sqrt{V_{tr}^2-\hat{P}_m^2/a^2}\right)}{\hat{P}_m^3} + \frac{1}{a\hat{P}_m\sqrt{V_{tr}^2-\hat{P}_m^2/a^2}}$$

$$\frac{dN_2}{d\hat{P}_m} = -\frac{\hat{P}_m}{a^3(V_{tr}^2-\hat{P}_m^2/a^2)^{(3/2)}}$$

$$\frac{d\,\mathrm{Den}}{d\hat{P}_m} = \frac{-2\frac{a^2\left(-b+\sqrt{V_{tr}^2-\hat{P}_m^2/a^2}\right)^2}{\hat{P}_m^3} - \frac{2\left(-b+\sqrt{V_{tr}^2-\hat{P}_m^2/a^2}\right)}{\hat{P}_m\sqrt{V_{tr}^2-\hat{P}_m^2/a^2}}}{\left(1+\frac{a^2\left(-b+\sqrt{V_{tr}^2-\hat{P}_m^2/a^2}\right)^2}{\hat{P}_m^2}\right)^2}$$

and then

$$\frac{d^2\delta_r}{d\hat{P}_m^2} = \frac{\left(-\frac{a\left(-b+\sqrt{V_{tr}^2-\hat{P}_m^2/a^2}\right)}{\hat{P}_m^2} - \frac{1}{a\sqrt{V_{tr}^2-\hat{P}_m^2/a^2}}\right)\left(-2\frac{a^2\left(-b+\sqrt{V_{tr}^2-\hat{P}_m^2/a^2}\right)^2}{\hat{P}_m^3} - \frac{2\left(-b+\sqrt{V_{tr}^2-\hat{P}_m^2/a^2}\right)}{\hat{P}_m\sqrt{V_{tr}^2-\hat{P}_m^2/a^2}}\right)}{\left(1+\frac{a^2\left(-b+\sqrt{V_{tr}^2-\hat{P}_m^2/a^2}\right)^2}{\hat{P}_m^2}\right)^2}$$

$$- \frac{2\frac{a\left(-b+\sqrt{V_{tr}^2-\hat{P}_m^2/a^2}\right)}{\hat{P}_m^3} + 1/a\hat{P}_m\sqrt{V_{tr}^2-\hat{P}_m^2/a^2} - \hat{P}_m/a^3(V_{tr}^2-\hat{P}_m^2/a^2)^{(3/2)}}{1+a^2\left(-b+\sqrt{V_{tr}^2-\hat{P}_m^2/a^2}\right)^2/\hat{P}_m^2}$$

$$\triangleq M_1 * M_2 * M_3 + M_4 * M_5 \tag{A3}$$

Here again we have split this equation such that M_i, $i = 1 \dots 5$ as well as its derivatives are defined as

$$M1 = \frac{1}{\left(1+\frac{a^2\left(-b+\sqrt{V_{tr}^2-\hat{P}_m^2/a^2}\right)^2}{\hat{P}_m^2}\right)^2}$$

$$M2 = -2\frac{a^2\left(-b+\sqrt{V_{tr}^2-\hat{P}_m^2/a^2}\right)^2}{\hat{P}_m^3} - \frac{2\left(-b+\sqrt{V_{tr}^2-\hat{P}_m^2/a^2}\right)}{\hat{P}_m\sqrt{V_{tr}^2-\hat{P}_m^2/a^2}}$$

$$M3 = -\frac{a\left(-b+\sqrt{V_t^2-\hat{P}_m^2/a^2}\right)}{\hat{P}_m^2} - \frac{1}{a\sqrt{V_t^2-\hat{P}_m^2/a^2}}$$

$$M4 = -\frac{1}{1+\frac{a^2\left(-b+\sqrt{V_{\mathrm{tr}}^2-\hat{P}_{\mathrm{m}}^2/a^2}\right)^2}{\hat{P}_{\mathrm{m}}^2}}$$

$$M5 = 2\frac{a\left(-b+\sqrt{V_{\mathrm{t}}^2-\hat{P}_{\mathrm{m}}^2/a^2}\right)}{\hat{P}_{\mathrm{m}}^3}+\frac{1}{a\hat{P}_{\mathrm{m}}\sqrt{V_{\mathrm{tr}}^2-\hat{P}_{\mathrm{m}}^2/a^2}}-\frac{\hat{P}_{\mathrm{m}}}{a^3(V_{\mathrm{t}}^2-\hat{P}_{\mathrm{m}}^2/a^2)^{(3/2)}}$$

and

$$\frac{\mathrm{d}M_1}{\mathrm{d}\hat{P}_{\mathrm{m}}} = -2\frac{-2\frac{a^2\left(-b+\sqrt{V_{\mathrm{tr}}^2-\hat{P}_{\mathrm{m}}^2/a^2}\right)^2}{\hat{P}_{\mathrm{m}}^3}-\frac{2\left(-b+\sqrt{V_{\mathrm{tr}}^2-\hat{P}_{\mathrm{m}}^2/a^2}\right)}{\hat{P}_{\mathrm{m}}\sqrt{V_{\mathrm{tr}}^2-\hat{P}_{\mathrm{m}}^2/a^2}}}{\left(1+\frac{a^2\left(-b+\sqrt{V_{\mathrm{tr}}^2-\hat{P}_{\mathrm{m}}^2/a^2}\right)^2}{\hat{P}_{\mathrm{m}}^2}\right)^3}$$

$$\frac{\mathrm{d}M_2}{\mathrm{d}\hat{P}_{\mathrm{m}}} = 6\frac{a^2\left(-b+\sqrt{V_{\mathrm{tr}}^2-\hat{P}_{\mathrm{m}}^2/a^2}\right)^2}{\hat{P}_{\mathrm{m}}^4}+\frac{6\left(-b+\sqrt{V_{\mathrm{tr}}^2-\hat{P}_{\mathrm{m}}^2/a^2}\right)}{\hat{P}_{\mathrm{m}}^2\sqrt{V_{\mathrm{tr}}^2-\hat{P}_{\mathrm{m}}^2/a^2}}+\frac{2}{(V_{\mathrm{tr}}^2-\hat{P}_{\mathrm{m}}^2/a^2)a^2}$$
$$-\frac{2\left(-b+\sqrt{V_{\mathrm{tr}}^2-\hat{P}_{\mathrm{m}}^2/a^2}\right)}{(V_{\mathrm{t}}^2-\hat{P}_{\mathrm{m}}^2/a^2)^{(3/2)}a^2}$$

$$\frac{\mathrm{d}M_3}{\mathrm{d}\hat{P}_{\mathrm{m}}} = 2\frac{a\left(-b+\sqrt{V_{\mathrm{tr}}^2-\hat{P}_{\mathrm{m}}^2/a^2}\right)}{\hat{P}_{\mathrm{m}}^3}+\frac{1}{a\hat{P}_{\mathrm{m}}\sqrt{V_{\mathrm{tr}}^2-\hat{P}_{\mathrm{m}}^2/a^2}}-\frac{\hat{P}_{\mathrm{m}}}{a^3(V_{\mathrm{tr}}^2-\hat{P}_{\mathrm{m}}^2/a^2)^{(3/2)}}$$

$$\frac{\mathrm{d}M_4}{\mathrm{d}\hat{P}_{\mathrm{m}}} = \frac{-2\frac{a^2\left(-b+\sqrt{V_{\mathrm{tr}}^2-\hat{P}_{\mathrm{m}}^2/a^2}\right)^2}{\hat{P}_{\mathrm{m}}^3}-\frac{2\left(-b+\sqrt{V_{\mathrm{tr}}^2-\hat{P}_{\mathrm{m}}^2/a^2}\right)}{\hat{P}_{\mathrm{m}}\sqrt{V_{\mathrm{tr}}^2-\hat{P}_{\mathrm{m}}^2/a^2}}}{\left(1+\frac{a^2\left(-b+\sqrt{V_{\mathrm{tr}}^2-\hat{P}_{\mathrm{m}}^2/a^2}\right)^2}{\hat{P}_{\mathrm{m}}^2}\right)^2}$$

$$\frac{\mathrm{d}M_5}{\mathrm{d}\hat{P}_{\mathrm{m}}} = -6\frac{a\left(-b+\sqrt{V_{\mathrm{tr}}^2-\hat{P}_{\mathrm{m}}^2/a^2}\right)}{\hat{P}_{\mathrm{m}}^4}-\frac{3}{a\hat{P}_{\mathrm{m}}^2\sqrt{V_{\mathrm{tr}}^2-\hat{P}_{\mathrm{m}}^2/a^2}}-\frac{3\hat{P}_{\mathrm{m}}^2}{a^5(V_{\mathrm{tr}}^2-\hat{P}_{\mathrm{m}}^2/a^2)^{(5/2)}}$$

The third derivative of δ_{r} with respect to $\hat{P}_{\mathrm{m}}$ is then given by

$$\frac{\mathrm{d}^3\delta_{\mathrm{r}}}{\mathrm{d}\hat{P}_{\mathrm{m}}^3} = \frac{\mathrm{d}M_1}{\mathrm{d}\hat{P}_{\mathrm{m}}}*M_2*M_3+M_1*\frac{\mathrm{d}M_2}{\mathrm{d}\hat{P}_{\mathrm{m}}}*M_3+M1*M_2*\frac{\mathrm{d}M_3}{\mathrm{d}\hat{P}_{\mathrm{m}}}+\frac{\mathrm{d}M_4}{\mathrm{d}\hat{P}_{\mathrm{m}}}*M_5+M_4*\frac{\mathrm{d}M_5}{\mathrm{d}\hat{P}_{\mathrm{m}}} \tag{A4}$$

Its complete expression being

$$\frac{\mathrm{d}^3\delta_r}{\mathrm{d}\hat{P}_m^3} = -\frac{-6\frac{a\left(-b+\sqrt{V_{tr}^2-\hat{P}_m^2/a^2}\right)}{\hat{P}_m^4} - \frac{3}{a\hat{P}_m^2\sqrt{V_{tr}^2-\hat{P}_m^2/a^2}} - \frac{3\hat{P}_m^2}{a^5(V_{tr}^2-\hat{P}_m^2/a^2)^{(5/2)}}}{1+\frac{a^2\left(-b+\sqrt{V_{tr}^2-\hat{P}_m^2/a^2}\right)^2}{\hat{P}_m^2}}$$

$$+\frac{2}{\left(1+\frac{a^2\left(-b+\sqrt{V_{tr}^2-\hat{P}_m^2/a^2}\right)^2}{\hat{P}_m^2}\right)^2} * \left(-2\frac{a^2\left(-b+\sqrt{V_{tr}^2-\hat{P}_m^2/a^2}\right)^2}{\hat{P}_m^3} - \frac{2\left(-b+\sqrt{V_{tr}^2-\hat{P}_m^2/a^2}\right)}{\hat{P}_m\sqrt{V_{tr}^2-\hat{P}_m^2/a^2}}\right)$$

$$* \left(2\frac{a\left(-b+\sqrt{V_{tr}^2-\hat{P}_m^2/a^2}\right)}{\hat{P}_m^3} + \frac{1}{a\hat{P}_m\sqrt{V_{tr}^2-\hat{P}_m^2/a^2}} - \frac{\hat{P}_m}{a^3(V_{tr}^2-\hat{P}_m^2/a^2)^{(3/2)}}\right)$$

$$-\frac{2\left(-2\frac{a^2\left(-b+\sqrt{V_{tr}^2-\hat{P}_m^2/a^2}\right)^2}{\hat{P}_m^3} - \frac{2\left(-b+\sqrt{V_{tr}^2-\hat{P}_m^2/a^2}\right)}{\hat{P}_m\sqrt{V_{tr}^2-\hat{P}_m^2/a^2}}\right)^2\left(-\frac{a\left(-b+\sqrt{V_{tr}^2-\hat{P}_m^2/a^2}\right)}{\hat{P}_m^2} - \frac{1}{a\sqrt{V_{tr}^2-\hat{P}_m^2/a^2}}\right)}{\left(1+\frac{a^2\left(-b+\sqrt{V_{tr}^2-\hat{P}_m^2/a^2}\right)^2}{\hat{P}_m^2}\right)^3}$$

$$+\frac{1}{\left(1+\frac{a^2\left(-b+\sqrt{V_{tr}^2-\hat{P}_m^2/a^2}\right)^2}{\hat{P}_m^2}\right)^2}\left[\left(6\frac{a^2\left(-b+\sqrt{V_{tr}^2-\hat{P}_m^2/a^2}\right)^2}{\hat{P}_m^4}\right.\right. \tag{A5}$$

$$\left.+\frac{6\left(-b+\sqrt{V_{tr}^2-\hat{P}_m^2/a^2}\right)}{\hat{P}_m^2\sqrt{V_{tr}^2-\hat{P}_m^2/a^2}} + \frac{2}{(V_{tr}^2-\hat{P}_m^2/a^2)a^2} - \frac{2\left(-b+\sqrt{V_{tr}^2-\hat{P}_m^2/a^2}\right)}{(V_{tr}^2-\hat{P}_m^2/a^2)^{(3/2)}a^2}\right)$$

$$\left.\left(-\frac{a\left(-b+\sqrt{V_{tr}^2-\hat{P}_m^2/a^2}\right)}{\hat{P}_m^2} - \frac{1}{a\sqrt{V_{tr}^2-\hat{P}_m^2/a^2}}\right)\right]$$

Now one may remark in (16) that we need the second and third derivatives of $\hat{P}_m$. These derivatives are not available, as they would imply the exact knowledge of P_m. To avoid this problem we first remember

$$\frac{\mathrm{d}\hat{P}_m}{\mathrm{d}t} = \gamma_1\tilde{\omega}_e$$

and remarking that

$$P_m = \theta = \tilde{\theta} + \hat{\theta}$$

one may compute

$$\frac{\mathrm{d}^2\hat{P}_{\mathrm{m}}}{\mathrm{d}t^2} = \gamma_1\dot{\tilde{\omega}}_{\mathrm{e}} = \gamma_1\frac{\omega_{\mathrm{s}}}{H}\tilde{P}_{\mathrm{m}} - \gamma_1\frac{D}{H}\tilde{\omega}_{\mathrm{e}}$$
$$= \gamma_1\frac{\omega_{\mathrm{s}}}{H}(P_{\mathrm{m}} - \hat{P}_{\mathrm{m}}) - \gamma_1\frac{D}{H}\tilde{\omega}_{\mathrm{e}}$$
$$= \gamma_1\frac{\omega_{\mathrm{s}}}{H}(\hat{\theta} - \hat{P}_{\mathrm{m}}) - \gamma_1\frac{D}{H}\tilde{\omega}_{\mathrm{e}} + \gamma_1\frac{\omega_{\mathrm{s}}}{H}\tilde{\theta}$$

and

$$\frac{\mathrm{d}^3\hat{P}_{\mathrm{m}}}{\mathrm{d}t^3} = \gamma_1\frac{\omega_{\mathrm{s}}}{H}\dot{\tilde{P}}_{\mathrm{m}} - \gamma_1\frac{D}{H}\dot{\tilde{\omega}}_{\mathrm{e}}$$
$$= \gamma_1\frac{\omega_{\mathrm{s}}}{H}(-\gamma_1\tilde{\omega}_{\mathrm{e}}) - \gamma_1\frac{D}{H}\left(\frac{\omega_{\mathrm{s}}}{H}\tilde{P}_{\mathrm{m}} - \frac{D}{H}\tilde{\omega}_{\mathrm{e}}\right)$$
$$= \left(\gamma_1\frac{D^2}{H^2} - \gamma_1^2\frac{\omega_{\mathrm{s}}}{H}\right)\tilde{\omega}_{\mathrm{e}} - \gamma_1\frac{D\omega_{\mathrm{s}}}{H^2}(\hat{\theta} - \hat{P}_{\mathrm{m}}) - \gamma_1\frac{D\omega_{\mathrm{s}}}{H^2}\tilde{\theta}$$

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