Abstract. This paper addresses the stability of time-delayed force-reflecting displays used in human-in-the-loop virtual reality interactive systems. A novel predictive haptic-device model-based approach is proposed. The developed solution is stable and robust, and does not require either the estimation of time delay or any knowledge on its behavior. It applies without any adaptations to constant or causal time-varying delays. Efforts have been focused to simple developments in order to make the approach easy to implement in commercial haptic libraries and build-in interface controllers. Althought this study focuses on virtual environments haptics, it can be easily spreaded to teleoperation1. The obtained results are presented and discussed.

Key Words. Virtual environment haptics, Varying time delayed control, stability and robustness.

1 INTRODUCTION

Virtual reality techniques, refer typically to human-in-the-loop or human centered advanced simulation or prototyping systems. The original feature of the concept lies in the multi-modality of the man-machine interaction, which involves all human sensory modalities. Among these capabilities, the haptic modality is of prime importance when it’s a matter to allow the human operator to experience honest manipulation and touching of virtual objects with realistic sensations of stiffness, roughness, temperature, shape, weight, contact forces, etc. These physical parameters are collected then interpreted by the human haptic modality through a direct touch and motion of, let say, human hand. Virtual environments are visually rendered to the human operator through screens, head mounted displays and other up-to-date advanced visual interfaces. Headphones are used to display 3D virtual sound. In the contrary to vision and auditory, haptic displays are active. Indeed, to render and display forces, the interfaces must be able to both constraint human desired motions and, to apply forces on the involved human part (e.g. hand). These interfaces are typically robotic devices that are capable: (i) to collect desired human motion or desired human applied force to be sent to the VE engine part of the simulation state update, and (ii) to display, to the human operator, subsequent virtual forces, computed thanks to computer haptics algorithms (collision detection, dynamic contact and reaction force computation, etc.). Applications of force reflection or force feedback are actually spreading to many domains. Among the well known ones: interactive surgical simulators, interactive driving simulators, interactive games, VE based teleoperation. A great demand is also experienced in virtual industrial prototyping. The last issue would extend to concurrent engineering and needs the potentiality to allow haptic interaction among a group of users sharing the same VE over a network. It is well known from physiological and psychological data of the haptic modality and from the haptic control theory that the haptic loop requires a high bandwidth of around 1 kHz to guarantee the stability of the haptic interaction, and more importantly, to make a coherent feedback between the visual and the haptic scenes. Developing a network protocol that can provide sufficient bandwidth with minimum latency to a group of distant users is a challenging problem [1] and physics-based models that simulate haptic interactions among users have begun to be developed [2]. Yet one of the important problems of haptic feedback, even if only one user interacts with the VE engine, is time delay. The difficult nature of some tasks, the lack of knowledge on
user abilities and behaviors, the problem of developing a universal controller for stable haptics could be also too complex. Obviously time delay during the transfer and processing of data may easily result in unstable forces and can be harmful to the user.

To the best knowledge of the authors, there is no work addressing the stability of VE delayed force feedback interaction, since in most known applications, the user is not distant from the interactive simulation engine. This paper proposes a simple and efficient solution to deal with this problem.

2 The linear case with constant time-delay

VE force reflecting techniques borrow a lot to the early teleoperation systems. Up-to-now, time delay is still known to be one of the most severe problems in force reflecting teleoperators. Many solutions have been proposed to deal with this problem. Some of the most attractive ones are based on passivisy derived from scattering network theory [3], [4], [5], and [6]. Others are based on a passive transformation of power parameters (namely velocity and force) into waves. Other techniques use known classical control theory to derive stable controller from Lyapunov criteria [7] and [8], we can state other works as [9] using \( \mu \)-synthesis and [10] with notion of virtual time delay, etc. Although Smith prediction method is known since 1957 [11], it has not been implemented in the early time-delayed force reflecting teleoperation systems. The reason that prohibits the use of Smith prediction approach lies in the practical impossibility to predict mainly (i) the remote environment behavior and (ii) the operator desired trajectories, since they are given on-line. Concerning virtual reality applications, since most controllers come from teleoperation experience [12] and [13], it was not surprising to notice that Smith prediction was not investigated as a potential solution for time-delayed VE haptic feedback controllers.

The originality of the proposed solution is in the some-how prediction of the master part within the remote part [14]. Hence, the developed equations lead to a scheme where only the master model appears and also the estimation of the time delay is necessary. The term “somehow prediction” is used to signify that in fact the proposed solution is not really a prediction since only the master model is required, which means that no prediction of operator behavior or trajectory is needed. However, the upwards and forwards time delays must be known.

Without loss of generality and to better understand the concept, a simple LTI model of a VE haptic interface is considered. Figure 1 shows the implementation of the proposed controller (colored part of the block diagram representation) within the haptic architecture. \( M(s) \) is the haptic device transfer function, \( s \) is the Laplace transform variable, \( E(s) \) is the VE transfer function (assumed continuous due to a high sampling frequency), \( x_m \), \( s_c \), and \( x_e \) are respectively master, virtual coupling and VE positions, \( F_e \) is the VE computed force, \( F_h \) is the operator applied force on the device, \( C(s) \) is the commonly used virtual coupling [15] and [16], which guarantee unconditional stability of the haptic interaction system in the absence of time-delay, finally, \( \tau_1 \) and \( \tau_2 \) are respectively upwards and forwards constant time delays. The closed loop transfer function of the haptic system without the proposed controller is given by:

\[
\frac{F_e(s)}{F_h(s)} = \frac{M(s)E(s)e^{-\sigma_1}}{1 + e^{-\sigma_1}e^{-\sigma_2}E(s)(M(s) + C(s))} \tag{1}
\]

This transfer function has an infinite eigenvalues as the time delay element is present in the characteristic equation. This may consequently imply an instability of the whole haptic interaction.

The proposed solution to overcome this instability is designed within the colored box of figure 1. The controller uses the process model of the haptic display it performs like a local feedback loop within the local remote environment (real or virtual). The resulting transfer function of the global system is a stable haptic feedback with a delayed input \( F_h \):

\[
\frac{F_e(s)}{F_h(s)} = \frac{M(s)E(s)e^{-\sigma_1}}{1 + E(s)(M(s) + C(s))} \tag{2}
\]

We can notice that, when using the proposed controller there is no more delay items in the characteristic equation of the closed loop system, equation 2. As stated before, the main advantage of this control-prediction scheme is in using the model of the haptic reflecting device only. The latter is well know and its parameters well identified. However, the controller requires to estimate both upwards and forwards time-delay.

![Figure 1: The master-model-based Smith prediction principle in the frame of a nominal LTI haptic feedback architecture.](image-url)
Obviously, this example can be easily generalized to any kind of haptic devices and mainly teleoperation systems. In this last case, \( E(s) \) represents the linear model of the remote system interacting with its environment.

3 The linear case with time-varying delay

As previously stated, the proposed method needs:

- a good knowledge of the master model, and
- the estimation of the time delay

Estimation of the model does not require complicated techniques. Well known model estimation methods (namely those developed in robotics) can be applied. Estimation of time delay is also easily made, especially when the delay is constant. Simple networking commands (such as a ping command) achieve the matter. Practically, the time delay may fluctuate since neither public or usual networking access protocols nor the computer haptics algorithms\(^2\) (in the frame of VE haptics) can guarantee time-determinism.

3.1 A more adequate practical implementation

When we look more the proposed controller (figure 1) we notice that in fact, the desired remote position is corrupted by a local closed loop on the obtained contact force. Indeed, the actual \( x_e \) is not directly the desired master position \( x_m \), but the delayed \( x_m \), minus the outputs of the local closed controller based on the master model, the delayed master model and the obtained contact force.

![Figure 2: A practical implementation of the controller.](image)

At the beginning, the controller have been implemented as it is. But from a practical and a simple observation, the structure of the controller makes possible an interesting extension which:

- avoids the estimation of time-delay, and
- makes a straightforward extension to time-varying time delay.
- unburden the VE from the buffering in order to compute the controller by sharing the computation on both sites.

This is obtained simply as depicted in figure 2. The controllers in the figures 1 and 2 are identical. Nevertheless, the second implementation highlights that it is no more necessary to estimate time delay, and more importantly: the behavior of time delay may have no effect on the stability of the system. The next section gives a more generic discussion and proves the given assertions.

The dynamic model of an haptic display can be approximated in a linear form\(^3\), considering an apparent mass \( M \) and friction \( B \):

![Figure 3: The actual implementation scheme.](image)

\[
M\ddot{x}_m = F_h + \tau
\]

A simple controller of the form: \( \tau(t) = -B\dot{x} - \alpha F_e(t - \tau_2(t)) \) leads to

\[
F_h(t) - \alpha F_e(t - \tau_2(t)) = M\ddot{x}_m + B\dot{x}_m
\]

where \( x_m, \dot{x}_m \), and \( \ddot{x}_m \) are respectively the Cartesian space position, speed and acceleration, \( F_h \) and \( F_e \) denote respectively human and VE forces applied to the haptic device, \( \alpha \) is a simple gain taken as 1 in this paper\(^4\). Time delays uses for the proof of the stability.

\(^2\)collision detection algorithms and dynamic force computation.

\(^3\)The developed proof holds for the non-linear haptic model described by the classical dynamic equation: \( M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) \). The demonstration is also trivial, we choose the linear case for the clarity of the presentation.

\(^4\)Parameters \( \alpha \) can be set to an adequate value to improve performances.
with the proposed control scheme are variable. The fundamental idea is to emulate a passive behavior of the haptic device and the transmission channel. In the second comparator of figure 3 we have \( x_h = x_m + x_e \), using the equality \( F_c(t - \tau_2(t)) = M\ddot{x}_e + B\ddot{x}_e \) are obtained:

\[
F_c(t - \tau_2(t)) = \ddot{M} [\ddot{x}_h - \ddot{x}_m] + \dddot{B} [\dddot{x}_h - \dddot{x}_m] \tag{4}
\]

Where \( \ddot{M} \) is the estimated apparent mass, \( \dddot{B} \) is the friction estimate. Equation 4 is used to cancel the effect the delayed control \( F_c(t - \tau_2(t)) \) from the haptic device (master) position \( x_m \) to be sent to the slave site.

Just after the transmission channel \( \tau_1(t) \), at the third comparator we have:

\[
F_e(t) = \ddot{M} [\ddot{x}_h(t - \tau_1(t)) - \ddot{x}_{mr}] + \dddot{B} [\dddot{x}_h(t - \tau_1(t)) - \dddot{x}_{mr}] \tag{5}
\]

where \( x_{mr} \) is the after-master position transmitted to the slave site. Equation 4 delayed by \( \tau_1(t) \) is substituted in equation 5 leads to:

\[
F_e(t - \tau_1(t - \tau_2(t))) - F_e(t) = \dddot{M} [\dddot{x}_{mr} - \dddot{x}_m(t - \tau_1(t))] + \dddot{B} [\dddot{x}_{mr} - \dddot{x}_m(t - \tau_1(t))] \tag{6}
\]

Finally, equation 3 is also delayed by \( \tau_1(t) \), and the obtained force \( F_c(t - \tau_1(t - \tau_2(t))) \) substituted in equation 6 leads to:

\[
F_h(t - \tau_1(t)) - F_e(t) = \dddot{M} \dddot{x}_{mr} + \dddot{B} \dddot{x}_{mr} + (M - \ddot{M}) \dddot{x}_m(t - \tau_1(t)) + (B - \dddot{B}) \dddot{x}_m(t - \tau_1(t)) \tag{7}
\]

If we assume that the estimation error of the apparent mass and friction is zero:

\[
\begin{cases}
M - \ddot{M} = 0 \\
B - \dddot{B} = 0
\end{cases}
\]

then equation 7 takes the following form:

\[
F_h(t - \tau_1(t)) - F_e = \dddot{M} \dddot{x}_{mr} + \dddot{B} \dddot{x}_{mr} \tag{8}
\]

This last equation exhibits a passive behavior of the equivalent new master side. The correction is equivalent to delay the input \( F_h \). Assuming that the virtual environment is passive, a fundamental property is that the feedback interconnection of passive systems is again passive [17], it ensues from it, that the haptic interaction is stable.

4 Simulation results

This section presents simulation results of the developed controller. The haptic display is a one DOF actuated arm with apparent mass \( m = 0.2kg \) and friction of about \( b = 3Ns/m \). The contact will be performed between the rigid virtual pen and a virtual walls of high stiffness \( K_w \). In this first simulation, time delays are taken constant but different, indeed \( \tau_1 = 1sec \) and \( \tau_2 = 0.5sec \). Figure 4 shows the tracking and force feedback behavior when the operator interacts with a VE of stiffness \( K_w \). It is assumed that collision detection and force computation are performed simply and do not cost additional time delay.

Figure 4 shows the result obtained of a simulating virtual contact. The operator applies a sinusoidal force \( F_h \) which drops the master and probe positions to increase until a contact is made between the probe and the wall. This is done when the virtual probe position reaches 20cm. From this time the local VE controller \( C(s) \) guarantees the local stability of the virtual interaction and the calculation of the virtual force to feedback to the operator. One can notice that when the contact is made, the master velocity \( v_m \) drops to zero and the fed force \( F_e \) (the controller) increases accordingly to \( F_h \) during the contact. The position discrepancy, when the contact is made, is unavoidable whatever is the controller or the approach (unless a very prediction is made in the master side), this is due to the undergo physical time-delay. Nevertheless, the virtual probe position \( x_e \) is stably maintained by the operator during the whole contact time. Several simulations are conducted with multiple hard and viscous contacts, they show that the behavior of the force feedback interaction is stable whatever the time delay. Obviously, one must not suspect that functional performances are acceptable for an actual use in the presence of important time delays.

Figure 5 shows the result of the previously set up system under similar parameters and time-varying delay. The variable time delay \( \tau_1(t) \) is represented on the same figure, it satisfies the causality constraints: \( t - \tau_1(t) > 0 \) and \( \dot{\tau}_1(t) < 1 \forall t \). In the simulation \( 0.5 \leq \tau_1(t) \leq 1.5 \text{ sec} \). Three different simulations are
performed: 1) time-varying \(\tau_1(t)\) with \(\tau_2(t)\) constant, 2) time-varying \(\tau_2(t)\) with \(\tau_1(t)\) constant and, 3) time-varying \(\tau_1(t)\) and \(\tau_2(t)\). Obtained simulation results show that \(\tau_2(t)\) dynamics has no effect on the stability of the overall force feedback system. This results make true the derived theoretical assumption stating that the adopted new implementation of the developed controllers cancels the effect of time delay. By analogy to the constant time delay case, since the time delays exponential are not present (after correction) in the characteristic equation. So in case 2, the simulation shows a stable behavior of the system, as if \(\tau_2(t)\) was constant i.e. the dynamic of \(\tau_2(t)\) is compensated by the controllers. For case 1 and 3, clearly the dynamic of \(\tau_1(t)\) affects the dynamic of the force reflecting system. Figure 5 shows clearly that in the transitions between two time delays behavior of \(\tau_1(t)\) i.e. varying and constant, the force reflecting response switches between two overall system corresponding behaviors. The transitions seem to be abrupt but do not affect the overall stability of the system. Similarly to the constant time delay linear case, this is related to the fact that the \(\tau_1\) exponential is still present in the closed loop transfer function numerator. Indeed the behavior of \(\tau_1(t)\) affects the behavior of force feedback but not its stability, as proven in theory, equation 8.

4.1 Robustness Analysis

Robustness analysis is performed in the case of constant time delay. The time delays has been approximated using Padé’s fourth order transfer function approximation, that is:

\[
\exp(-s\tau) \approx \lim_{n \to \infty} \left( \frac{1 - \frac{s\tau}{n}}{1 + \frac{s\tau}{n}} \right)^n
\]

A root locus of the closed loop transfer function has been performed by varying the controller’s parameters that is to say. Figure 6 shows that within a determined margins, force refection is still stable although some light oscillations appear in the master position \(x_m\) and the reflected force \(F_e\). In this simulation in figure 6 \(m = 0.3\ kg\) and \(b = 5\ Nm/s\) are the estimated parameters, we can notice that the behavior of the interaction is still stable, figure 7.

5 Conclusion

The paper presents a master-model based controller designed to stabilize delayed force feedback systems. The proposed method is based on an astute implementation of a somehow Smith prediction scheme.
which requires only the haptic device model and do not necessitate the estimation of both (upwards and downwards) delays. Simulation results confirm a stable force reflection from the VE in presence of constant and also time-varying delays. A robustness analysis of the proposed controller has also been conducted. The error margins that guarantee the stability of fed back forces are found to be wide enough to allow using a linear model of the haptic interface based only on a apparent mass and friction estimation.

Comparing to wave-based approaches, the proposed solution is more transparent to the user, since there is no additional corrupting damping as engendered from the transformation of force and flux parameters into waves. The price to be paid is in the position discrepancy between the master and the virtual avatar when the contact is made which may be more important in our controller case comparing to wave-based methods. In fact, in wave-based method, the artificial damping increases with speed (in free motion), which prevents important master-slave position discrepancies, but there is an additional force felt which is not directly related to actual remote contact forces, that is to say more stable but less transparent.

Future work is focused in improving performances in virtual environment haptics. A prediction within the master site is possible based on computer haptics algorithms.

REFERENCES


